

ESSAYS IN FINANCIAL ECONOMETRICS AND QUANTITATIVE
INDUSTRIAL ORGANIZATION

by

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B.S., Amirkabir University (Tehran Polytechnic), Iran, 2002

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AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the
requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Economics
College of Arts and Sciences

KANSAS STATE UNIVERSITY
Manhattan, Kansas

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Abstract

This dissertation consists of one essay in financial econometrics and two essays in quantitative industrial organization. The first essay studies the relationship between stock return volatility and current and prior shocks to oil price volatility. We study the behavior of aggregate stock markets as well as individual industry sectors. Our results show that lagged stock return volatility is the main determinant of current stock return volatility in aggregate markets, with oil price volatility providing no additional information that can be used to forecast stock return volatility. For individual industry sectors, we find a robust and stable prediction relationship only for the chemicals industry. Additional estimation exercises confirm the robustness of these results.

The second essay uses a Bertrand-Nash price-competition framework to model a vertically integrated provider (VIP) that is a monopoly supplier of an essential input for downstream production. An input price that is “too high” can lead to inefficient foreclosure and one that is “too low” creates incentives for nonprice discrimination. The range of non-exclusionary input prices is circumscribed by the input prices generated on the basis of upper-bound and lower-bound displacement ratios. The admissible range of the ratio of downstream to upstream “price-cost” margins for the VIP is increasing in the degree of product differentiation and reduces to a single ratio in the limit as the products become perfectly homogeneous.

The third essay explores the relationship between upstream input prices and downstream market exclusion under a Stackelberg quantity-competition framework. Market exclusion is a concern when input prices are “too high” and “too low” because it can result in inefficient foreclosure and sabotage, respectively. Consistent with the results obtained in the second essay, the safe harbor range of downstream to upstream “price-cost” margin ratios is decreasing in the degree of product homogeneity and approaches a single ratio in the limit as the prod-

ucts become perfectly homogeneous. This single margin ratio preserves equality between the VIP's wholesale and retail "price-cost" margins. A key finding for competition policy is that the bounds of non-exclusionary input prices are markedly wider under Bertrand-Nash competition than they are under Stackelberg competition. Hence, it is critical that the antitrust and regulatory authorities understand the nature of the industry competition so that rules governing permissible conduct are properly calibrated to yield efficient outcomes.

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¹This chapter is the extended version of a published paper (see [Nadimi & Weisman, 2014](#)).

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Dedication

To my amazing wife, Shiva, for her endless love and support.

Chapter 1

Stock Market Return Volatility and Oil Price Volatility

1.1 Introduction

Before 1973, the price of oil was fairly stable (see Figure [1.1](#)). The U.S. Seven Sisters oil companies stabilized the nominal oil price through production and price controls during most years of the 20th century. After the Yom Kippur War started on October 6, 1973, the increasing influence of OPEC on the crude oil market caused oil prices to start behaving like prices of other commodities ([Driesprong et al., 2008](#)).

Oil shocks have long been considered an important determinant of GDP. [Adelman \(1993, p. 537\)](#) states that,

“Oil is so significant in the international economy that forecasts of economic growth are routinely qualified with the caveat: ‘Provided there is no oil shock.’”

Estimates by the International Monetary Fund ([Mussa, 2000](#)) imply that a \$5/barrel oil price increase will cause the global economic growth rate to fall by 0.3 percent the following year. In contrast to the volume of studies on the relationship between oil price changes and the economy (see for example [Mork, 1989](#); [Lee et al., 1995](#); and [Hamilton, 2003](#), among many

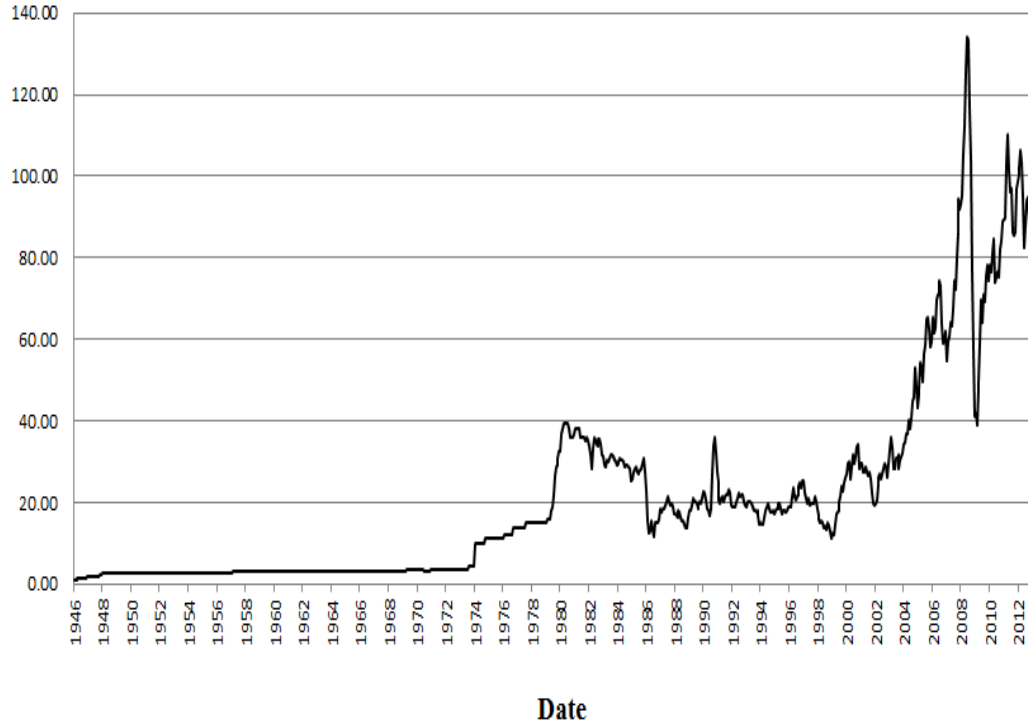


Figure 1.1: West Texas Intermediate oil price (US\$/Barrel) from 1946 to 2012. Data are obtained from Federal Reserve Economic Data (FRED).

others), fewer studies have investigated the relationship between oil prices and stock returns. Surprisingly, the findings of these studies show almost no consensus among economists about the link between stock prices and the oil prices.

Using a bivariate vector autoregression (VAR) model, [Kling \(1985\)](#) studies oil shocks and US stock market behavior and concludes that an increase in crude oil prices reduces future stock prices in industries that use oil as input factors. In contrast, using the Arbitrage Pricing Theory (APT), [Chen *et al.* \(1986\)](#) find no evidence for the influence of oil price changes on asset prices in U.S. Using a standard cash-flow dividend valuation model with quarterly data, [Jones & Kaul \(1996\)](#) test whether the international stock market reactions to oil shocks can be justified by current and future fluctuations in real cash flows and/or expected returns. They report that real oil price increases have a significant negative effect on U.S., Canadian, Japanese, and UK aggregate stock returns. [Huang *et al.* \(1996\)](#) investigate

the relationship between daily oil futures returns and daily US stock returns using a vector autoregression model. They find that changes in the price of oil futures affect stock returns of some individual oil companies, but they do not predict future aggregate stock returns like the S&P 500. [Sadorsky \(1999\)](#) uses a vector autoregression model with four variables including industrial production, interest rates, real oil prices, and real stock returns and finds that oil price changes play an important role in stock market fluctuations. [Driesprong *et al.* \(2008\)](#) find that an increase in oil prices drastically lowers future stock returns worldwide, in both developed and emerging markets. They also show that the relationship between monthly stock returns and lagged monthly oil price changes is strengthened when more lagged values of oil price changes are included. [Kilian & Park \(2009\)](#) find that demand and supply shocks in the oil market have different influences on U.S. real stock returns, and they jointly account for 22 percent of the long-run variation in U.S. real stock returns.

In light of the literature on the relationship between oil price movements and stock returns, it is natural to ask whether oil price volatility has predictive power for stock return volatility. While prediction of stock returns using lagged oil prices may violate the principle of market efficiency (see [Driesprong *et al.*, 2008](#)), there is no theoretical reason that stock return volatility should not be predictable, even in a market where stock prices fully reflect all publicly available information.

To the best of our knowledge, the relationship between oil price volatility and stock return volatility has received little attention in the literature, and the related studies are limited in number. Using monthly data on industrial production, interest rates, oil prices, and stock prices, [Sadorsky \(1999\)](#) estimates a vector autoregression model and finds that oil price volatility significantly affects U.S. real stock returns. He also finds evidence of an asymmetric effect of oil price volatility shocks on real U.S. stock returns. [Sadorsky \(2003\)](#) finds that the conditional volatility of oil price, the term premium, and the consumer price index each have significant impacts on the conditional volatility of technology stock prices. [Hammoudeh *et al.* \(2004\)](#) use a GARCH model to examine the effect of crude oil market

volatility on the equity return volatility of the S&P oil sector stock indices. They find that the volatility of oil futures have a resonant effect on the volatility of the stock returns in oil and gas exploration, production, and domestic integrated oil companies. Utilizing a GARCH(1,1) technique, [Elyasiani *et al.* \(2011\)](#) find that oil return volatility has a strong impact on excess stock return volatility in oil-related and oil-substitute industries, but they do not see such predictability in other industries.

Using data for the S&P 500, CRSP aggregate markets, and fifteen individual industry sectors, along with data for several measures of oil prices, including West Texas Intermediate (WTI), the U.S. Producer Price Index (PPI) for crude oil and the Refiners' Acquisition Cost (RAC) for imported crude oil, this essay investigates whether oil price volatility predicts stock return volatility. Our analysis covers the time period from October 1973 to December 2012.

This essay makes four important contributions to the literature. The first, and most important, is that of testing for stock return volatility predictability. If stock returns can be predicted by lagged oil price changes, as has been claimed in some of the literature discussed above, it is plausible that stock return volatility can be predicted using lagged oil price volatility. Second, we want to see how the relationship between oil price volatility and stock return volatility has changed, if at all, through time. In other words, to forecast stock return volatility in practice, it is necessary that the relationship is stable through time. Third, we compare conditional return volatilities constructed from GARCH models with realized return volatilities. Fourth, in contrast to the few related existing studies that rely only on GARCH models to forecast volatility, we use an ordinary least squares (OLS) model that provides results consistent with the methods used in the finance literature.

As mentioned earlier, for our investigation, we use two different measures of return volatility including realized return volatility (measured as the standard deviation of daily natural log returns over the course of a month) and conditional return volatility (measured as GARCH(1,1) fitted values of monthly natural log returns). This measure is sometimes called

the conditional standard deviation. Our findings show that these two return volatility measures provide similar results. In contrast to the findings of existing studies that a volatile oil market is followed by future stock return volatility, our in-sample estimation results indicate that oil price volatility does not have forecast power for future aggregate stock return volatility, and the current behavior of the stock return volatility is mainly explained by its prior fluctuations. Our study distinguishes itself from the existing studies in that it examines the forecasting relationship between oil price volatility and stock return volatility on a monthly basis over different time periods. Also, variety of robustness checks confirm that our findings are not an artifact of a particular choice of model specification and data series over a specific time period. The existing studies either examine the forecasting relationship on a daily bases (see [Hammoudeh *et al.*, 2004](#) and [Elyasiani *et al.*, 2011](#)) or are limited to specific data series and time period without considering the effect of prior values of stock return volatility on its current values (see [Sadorsky, 2003](#)).

Our findings are largely robust to the use of other oil price series. Although the volatility of the U.S. producer price index for crude oil shows predictive power for S&P 500 return volatility over the period 1986:1-2012:12, this is the only evidence of predictability, and it is not stable across other time periods. We do find evidence of a contemporaneous relationship between oil price realized volatility and stock return realized volatility after 1986. That does not hold when we use the conditional return volatility series generated by a GARCH(1,1) model. The motivation for introducing realized volatility was to use all of the available data when constructing a measure of the volatility of a time series (see [Andersen & Bollerslev, 1998](#) and [Andersen *et al.*, 2003](#)). A possible explanation for finding a contemporaneous relationship between the two volatility series only when using realized volatility measures is that the GARCH(1,1) model discards most of the available data when making a monthly volatility prediction. This suggests that related future studies should not rely on only GARCH models. For individual industry sectors, we find evidence of predictability in four out of fifteen industries including chemicals, coal, rubber, and construction, but the relationship is

stable only for the chemicals industry.

We check the robustness of our results to changes in the methodology in several ways. A GARCH(1,1)-X specification that includes the lagged variance of oil price changes as an exogenous regressor in the stock return variance equation delivers similar conclusions on the absence of predictability. Impulse response function analysis finds no statistically significant reaction of stock return volatility to an oil price volatility shock. Rolling-window estimates of the prediction models uncover evidence of extreme time variation in the estimated parameters of the models that we estimate. An evaluation of out-of-sample stock return volatility forecasts is consistent with the in-sample analysis - there is no advantage to predicting stock return volatility using oil price volatility. We conclude that there is no evidence that oil price volatility is a useful predictor for stock return volatility.

The remainder of this chapter is organized as follows. In Section 1.2 we discuss our data. Section 1.3 describes the empirical methodology. Section 1.4 discusses the baseline empirical results and the robustness tests. Section 1.5 contains concluding remarks.

1.2 Oil and stock market data

1.2.1 Oil price data

The crude oil market is the largest commodity market in the world. Between 70 and 80 million barrels of crude oil are produced daily worldwide, almost 25 percent of which is consumed by the United States. Crude oil is traded daily on spot, futures, and over-the-counter markets mostly at the New York Mercantile Exchange (NYMEX) and the International Petroleum Exchange (IPE) in London (Levin *et al.*, 2003).

Spot markets exist for different oil grades and are located in different regions (for example, Rotterdam/Northwest Europe, Singapore/South East Asia, and Cushing, Oklahoma/U.S. Gulf Coast). Most of them focus on prompt delivery of readily available supplies. For a regional market to develop into a pricing center, some foundation in logistics are required

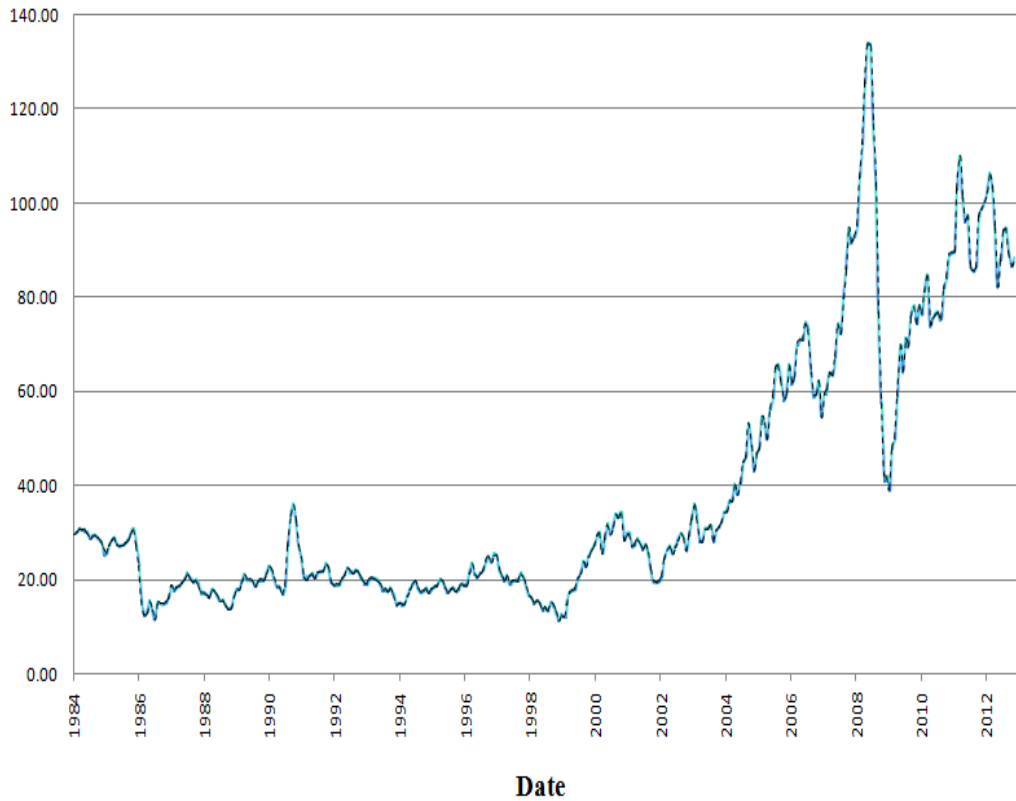


Figure 1.2: Oil price development over the period 1984:1-2012:12 for West Texas Intermediate spot prices (black dotted line) and one month futures prices (blue solid line). Data are obtained from the U.S. Energy Information Administration (EIA).

such as ready supply, storage facilities, choices of transportation, and many sellers and buyers. Spot prices in these different markets are reported by a variety of sources. There are also futures markets for oil. In a futures contract, the seller agrees to deliver a given amount of a commodity at a specified price, place, and future month ([Driesprong et al., 2008](#)).

Spot oil prices and oil futures prices tend to move closely together although these prices are usually slightly different as shown in Figure 1.2.

Among different grades of crude oil, West Texas Intermediate, also known as Texas light sweet, is often used as a benchmark for crude oil prices. West Texas Intermediate is light because of its low density, and sweet due to its low sulfur content. Brent and Dubai are other commonly cited oil prices. Generally the price of a lighter and sweeter oil grade is higher, as it is cheaper to process. Since West Texas Intermediate is lighter and sweeter than Brent oil,

it is generally more expensive than Brent oil (and considerably more expensive than Dubai). Cushing, Oklahoma, is the most active transport hub for West Texas Intermediate since it has many intersecting pipelines, storage facilities and ready supply ([Driesprong *et al.*, 2008](#))

Empirical studies may be sensitive to the choice of oil price measure. Beside West Texas Intermediate crude oil, the U.S. producer price index for crude oil, and the U.S. refiners' acquisition cost for imported crude oil, for domestic crude oil, and for a composite of domestic and imported crude oil are the most frequently used candidates for the oil price series in the literature. However, there is no general consensus among researchers on which oil price to use ([Kilian & Vigfusson, 2011](#)). For instance, [Chen *et al.* \(1986\)](#), [Jones & Kaul \(1996\)](#), [Sadorsky \(1999\)](#), and [Hamilton \(2003\)](#), among others have used the U.S. producer price index for crude oil, and [Mork \(1989\)](#), [Kilian \(2009\)](#), and [Kilian & Vigfusson \(2011\)](#), among others have employed the refiners' acquisition cost for imported crude oil as a measure of oil prices in their studies. [Hamilton \(2011\)](#) claims that when the goal is to test for asymmetries in the shock transmission from oil price movements to U.S. real GDP, the U.S. producer price index for crude oil is a more accurate proxy for imported crude oil than the refiners' acquisition cost, as it is more strongly correlated with the price of gasoline. [Kilian & Vigfusson \(2011\)](#) argues that the logical conclusion of Hamilton's reasoning is that we should always use the retail price of gasoline rather than the price of oil since it is a good proxy for the retail price of energy for consumers and firms. However, Hamilton's reasoning is the basis for the methodology in the empirical studies by [Edelstein & Kilian \(2009\)](#), [Hamilton \(2009\)](#), and [Ramey & Vine \(2010\)](#).

[Mork \(1989\)](#) argued that the U.S. producer price of crude oil is not a good proxy for the price of oil paid by firms. He argued that this oil price was controlled by the government for much of the sample and as such may not reflect the true market price. [Mork \(1989, p. 741\)](#) states that

“...during the price controls of the 1970s, this index is misleading because it reflects only the controlled prices of domestically produced oil. However, since

the price control system closely resembled a combined tax/subsidy scheme for domestic and imported crude oil, the marginal cost of crude to U.S. refiners can be approximated by the composite (for domestic and imported) refiner acquisition cost for crude oil.”

Hence, it is unlikely that there is one price of oil that is appropriate for all purposes. [Hamilton \(2011\)](#) suggests to check the sensitivity of the results using alternative oil price measures. If similar results are obtained using different oil price measures, the results are more reliable. Of course, a failure to find similar results across oil price series would not be surprising, as it may reflect the measurement error in one or more of the series.

Our oil price data are daily and monthly spot prices of West Texas Intermediate. The main reason to use West Texas Intermediate for our analysis is because it is known as a benchmark for crude oil prices in the United States. West Texas Intermediate is the most important and liquid market for crude oil in the United States and most of the domestic crude oil grades are typically priced against its calendar monthly average. While Brent and many other crude oil types are waterborne cargo markets, West Texas Intermediate is a pipeline market where crude oil flows at near-constant rates into its mid-country delivery point located in Cushing, Oklahoma. The other reason is that West Texas Intermediate, to the best of our knowledge, has the longest available daily series for crude oil spot prices starting from January 2, 1986. We use the daily West Texas Intermediate oil spot prices from January 2, 1986 to December 31, 2012. For monthly data, our time period is from October 1973, at the beginning of Yom Kippur War in October 1973 when oil prices started to fluctuate, to December 2012. Hence, we base our results on 6812 daily observations and 471 monthly observations. West Texas Intermediate oil price series are available in Federal Reserve Economic Data (FRED) website. We have used the natural log returns ($\ln \frac{P_t}{P_{t-1}}$) of daily and monthly West Texas Intermediate spot prices.

In order to show the robustness of our results to the use of other oil price series, we also consider the monthly U.S. producer price index for crude oil favored by Hamilton and the

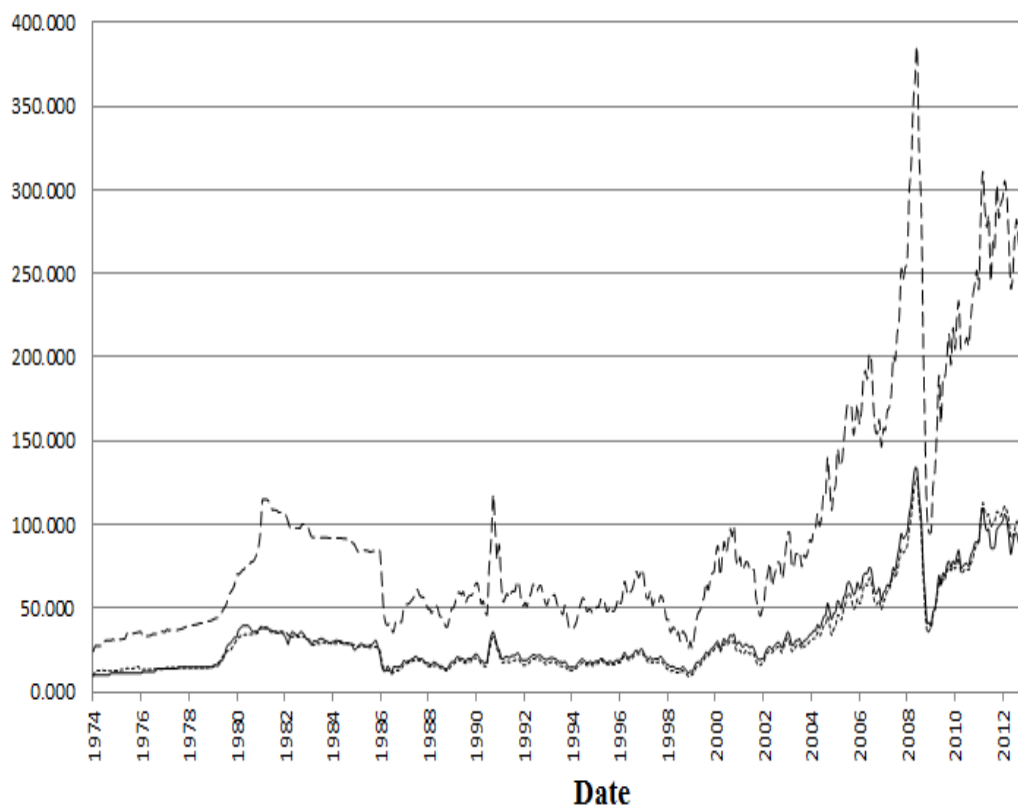


Figure 1.3: West Texas Intermediate (solid line), refiners' acquisition cost (dotted line), and the U.S. producer price index for crude oil (dashed line) fluctuations over the period 1974:1-2012:12. Data are obtained from the U.S. Energy Information Administration (EIA).

refiners' acquisition cost for imported crude. These oil price series are not available daily. The monthly refiners' acquisition cost for imported crude oil is available starting in January 1974. Both of these oil price series cover the period from January 1974 to December 2012 and are obtained from the U.S. Energy Information Administration (EIA). Natural log returns of the U.S. producer price index for crude oil and the refiners' acquisition cost for imported crude oil are used. Even though price differences do exist, our oil prices series tend to follow similar trends over time, as shown in Figure 1.3.

1.2.2 Stock market data

For our investigation we calculate daily and monthly log returns of both the Standard and Poor's 500 (S&P 500) stock market index and the Center for Research in Security Prices

(CRSP) value-weighted market portfolio. S&P 500 stock prices and CRSP value-weighted returns data are obtained from the Federal Reserve Economic Data (FRED) and Kenneth French's websites, respectively.¹ We use CRSP value-weighted portfolio since the value-weighted indices exhibit less autocorrelation relative to equally-weighted indices. Moreover, value-weighted indices are less affected by the January effect, as the January effect is highly related to the small firm effect (see [Hawawini & Keim, 1995](#)). Furthermore, daily and monthly value-weighted returns for fifteen oil sensitive industry sectors are obtained from Kenneth French's website.² Our industry-level data are constructed from the CRSP database and therefore are consistent with our aggregate CRSP stock returns data. The investigated industries are: Petroleum and Natural Gas, Automobiles and Trucks, Retail, Precious Metals, Chemicals, Rubber and Plastic Products, Construction, Steel works, Machinery, Aircraft, Shipbuilding and Railroad Equipment, Coal, Utilities, Banking, and Insurance. Daily and monthly log returns on our industry-level data are also calculated. Daily data series start on January 2, 1986 and end on December 31, 2012. Monthly data cover the period 1973:10-2012:12.

Return Volatility

For our investigation, we use two methods for calculating the volatility of the stock return and oil price series:

Realized return volatility

Our measure of realized volatility is the sample standard deviation, which has been extensively used as a measure of volatility in the existing literature (see [French *et al.*, 1987](#); [Schwert, 1989](#); [Fleming, 1998](#); [Christensen & Prabhala, 1998](#); [Andersen *et al.*, 2003](#); and [Bandi & Perron, 2006](#), among others). To construct monthly realized return volatility, we

¹See <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/datalibrary.html>.

²See <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/datalibrary.html>. We use the file containing 49 industry portfolios.

compute the standard deviation of daily natural log returns over the course of a month. In this study, monthly realized return volatility of both oil and stock markets are used. An advantage of the realized volatility measure relative to other methods, including GARCH models, is that one is able to use all of the daily observations to construct a measure of monthly volatility. To our knowledge, no published papers have looked at the relationship between stock return volatility and oil price volatility using realized volatility measures.

Conditional return volatility

It is common to construct estimates of conditional volatility using low order generalized autoregressive conditional heteroskedastic (GARCH) model (see [Bollerslev, 1986](#) and [Bollerslev et al., 1992](#)). A univariate regression with GARCH(p, q) error process can be represented as

$$\varepsilon_t = \sqrt{h_t} z_t, \quad \varepsilon_t \mid \psi_{t-1} \sim N(0, h_t), \quad t = 1, \dots, T \quad (1.1)$$

and

$$h_t = \omega + \sum_{i=1}^q \gamma_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \varphi_j h_{t-j}, \quad (1.2)$$

where $z_t \sim iid(0, 1)$, ε_t is a stochastic process, and ψ_{t-1} is the information set available at time $t - 1$. For a GARCH(1,1) model, the conditional variance is defined as

$$h_t = \omega + \gamma_1 \varepsilon_{t-1}^2 + \varphi_1 h_{t-1}. \quad (1.3)$$

We fit a univariate GARCH(1,1) model to the monthly log returns of West Texas Intermediate spot prices and monthly stock log returns to construct monthly conditional volatility (conditional standard deviation) of oil returns and stock returns, respectively.³

³[Bollerslev et al. \(1992\)](#) recommend using a low order GARCH model, particularly GARCH(1,1).

Table 1.1: GARCH(1,1) estimates for West Texas Intermediate monthly log returns over the period 1973:10-2012:12.

Parameter	Estimate	<i>t</i> -value
ω	0.0008	1.00
γ_1	0.102	0.90
φ_1	0.754	5.57

Table 1.1 shows the GARCH(1,1) estimates for West Texas Intermediate monthly log returns over the period 1973:10-2012:12. $\gamma_1 + \varphi_1 = 0.856$ denotes that the conditional variance of West Texas Intermediate log returns is very persistent, indicating that a shock to its price changes variance decays slowly. Bollerslev (1986) shows that in the univariate GARCH model defined by equation (1.2), $\sum_{i=1}^q \gamma_i + \sum_{j=1}^p \varphi_j < 1$ is a sufficient condition for strict covariance stationarity. Nelson (1990) shows that in a GARCH(1,1) model, when $\omega > 0$, the conditional variance is strictly stationary if and only if $E[\ln(\gamma z_t^2 + \varphi_1)] < 0$. In a simple GARCH(1,1) model with $z_t \sim N(0, 1)$, an easy application of Jensen's inequality shows

$$E[\ln(\gamma z_t^2 + \varphi_1)] < \ln[E(\gamma z_t^2 + \varphi_1)] = \ln(\gamma_1 + \varphi_1), \quad (1.4)$$

which implies that a GARCH(1,1) model with $\omega > 0$, is strict stationary if $\gamma_1 + \varphi_1 \leq 1$ and is strictly covariance stationary if $\gamma_1 + \varphi_1 < 1$ (see Bollerslev *et al.*, 1994 for related discussions). This indicates that the conditional West Texas Intermediate price volatility is strictly both covariance stationary and variance stationary.

Table 1.2 reports the summary statistics of monthly price volatility for West Texas Intermediate. Both realized and conditional West Texas Intermediate price volatility series are significantly skewed and show significantly high kurtosis, indicating that the extreme values are prevalent across West Texas Intermediate price volatility series. We reject the null hypothesis of normal distribution based on the Jarque-Bera (J-B) test statistics, and the Ljung-Box (L-B) Q-statistic reveals the presence of first order serial correlation in both realized and conditional oil price volatility series. The Augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) test statistics indicate that the null hypothesis of unit root is rejected

Table 1.2: Basic characteristics of monthly West Texas Intermediate realized price volatility over the period 1986:1-2012:12 and conditional price volatility over the period 1973:10-2012:12. We test the null hypothesis of no skewness using D’Agostino test. To test kurtosis, we use Anscombe-Glynn test with the null hypothesis of kurtosis is equal to three to detect whether there is a significant difference from the kurtosis of normally distributed data. The critical values for ADF and PP at the 1%, 5%, and 10% levels are -3.44, -2.87, and -2.57, respectively. $\rho(1)$ denotes the coefficient in an autoregressive regression of order one (AR(1)) on West Texas Intermediate price volatility. * and ** indicate significant p -values at 5% and 1% level, respectively. Pair-wise correlation over the period 1986:1-2012:12 is also reported.

West Texas Intermediate	Realized volatility	Conditional volatility
Obs.	324	471
Mean	0.022	0.079
Maximum	0.109	0.293
Minimum	0.007	0.060
Skewness	2.48**	4.02**
Kurtosis	12.98**	25.90**
L-B	104.46**	348.88**
ADF	-5.55	-7.09
PP	-9.76	-6.13
J-B	1679.32**	11564.78**
$\rho(1)$	0.56**	0.85**

Pairwise correlation between West Texas Intermediate price realized and conditional volatility is 0.61.

across both realized and conditional oil price volatility series at one percent level. Highly significant $\rho(1)$ also confirm the results of Ljung-Box test. However, the results of higher order West Texas Intermediate volatility AR regressions (not reported) showed that only up to two-month lagged oil price volatility is significant, indicating that a shock to West Texas Intermediate price volatility series decays quickly.

Table 1.3 shows the GARCH(1,1) estimates for S&P 500 and CRSP monthly log returns over the period 1973:10-2012:12. $\gamma_1 + \varphi_1$ equal to 0.932 and 0.956 in S&P 500 and CRSP aggregate markets, respectively show that the conditional variances of aggregate stock returns are persistent, indicating that a shock to their returns variance decays slowly, but the stock return volatility series are still strictly stationary. Table 1.4 presents the summary statistics of monthly realized and conditional return volatility in two aggregate markets including S&P 500 and CRSP. All market return volatility series show significantly high skewness and kurtosis. The Jarque-Bera test statistics, and the Ljung-Box Q-statistic reject the null of

Table 1.3: GARCH(1,1) estimates for S&P 500 and CRSP monthly log returns over the period 1973:10-2012:12.

Parameter	S&P 500		CRSP	
	Estimate	<i>t</i> -value	Estimate	<i>t</i> -value
ω	0.0001	1.36	0.0001	1.16
γ_1	0.179	2.55	0.103	2.28
φ_1	0.753	8.85	0.853	17.97

Table 1.4: Basic characteristics of monthly stock realized return volatility over the period 1986:1-2012:12 and conditional return volatility over the period 1973:10-2012:12 for S&P 500 and CRSP aggregate markets. We test the null hypothesis of no skewness using D'Agostino test. To test kurtosis, we use Anscombe-Glynn test with the null hypothesis of kurtosis is equal to three to detect whether there is a significant difference from the kurtosis of normally distributed data. The critical values for ADF and PP at the 1%, 5%, and 10% levels are -3.44, -2.87, and -2.57, respectively. * and ** indicate significant *p*-values at 5% and 1% level, respectively.

Market	Realized volatility									
	Obs.	Mean	Min	Max	Skew	Kurt	L-B	ADF	PP	J-B
S&P 500	324	0.010	0.003	0.061	3.49**	22.05**	128.37	-4.32	-8.81	5561.14**
CRSP	324	0.009	0.002	0.052	3.05**	16.89**	139.68**	-4.34	-8.57	3109.48**

Market	Conditional volatility									
	Obs.	Mean	Min	Max	Skew	Kurt	L-B	ADF	PP	J-B
S&P 500	471	0.037	0.022	0.108	2.09**	9.44**	371.81**	-4.45	-5.25	1158**
CRSP	471	0.046	0.030	0.095	1.18**	4.49**	405.59**	-3.80	-4.39	184.73**

normal distribution and reveal the presence of first order serial correlation, respectively, in both investigated markets. The Augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) test statistics exceed the critical values at the one percent level in all cases. Hence, the null hypothesis of unit root is also rejected across all markets.

Table 1.5 reports the results of the AR estimation equation (1.5) of order up to three for both S&P 500 and CRSP aggregate markets.

$$rvol_t^{market} = \mu_1 + \sum_i \beta_i rvol_{t-i}^{market} + \varepsilon_t, \quad (1.5)$$

where $rvol_t^{market}$ denotes stock return volatility. The results indicate that a shock to the stock market return volatility dies out very fast, and higher than first order autocorrelations tend to be small and insignificant. However, Breusch-Godfrey test results (not reported)

Table 1.5: Estimation results of equation (1.5) for S&P 500 and CRSP aggregate markets. We report the results over the periods 1986:1-2012:12 for realized return volatility series and 1973:10-2012:12 for conditional return volatility series. * and ** indicate significant coefficients at the 5% and 1% level, respectively, based on White standard errors.

Coefficients	Realized volatility					
	S&P 500			CRSP		
	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
β_1	0.63**	0.52**	0.50*	0.63**	0.54**	0.53**
β_2		0.16	0.12		0.16	0.11
β_3			0.09			0.08
	Conditional volatility					
	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
β_1	0.88**	0.93**	0.93**	0.92**	0.96**	0.96**
β_2		-0.05	-0.11		-0.03	-0.06
β_3			0.06			0.02

show the presence of autocorrelation of orders higher than one in the error term of the AR(1) regressions on realized stock return volatility in both S&P 500 and CRSP aggregate markets. For the AR(1) regressions on conditional market return volatility, the Breusch-Godfrey test does not indicate the presence of autocorrelation of orders higher than one. Based on these basic characteristics, when justified, we use heteroskedasticity consistent or White standard errors in the remainder of our analysis that also adjust for the high level of kurtosis (see [White, 1980](#)). However, when Breusch-Godfrey test shows the presence of autocorrelation of orders higher than one, heteroskedasticity and autocorrelation consistent or Newey West standard errors are used.

1.3 Basic regression model

To investigate the interaction between oil price volatility and stock return volatility, we estimate a linear regression by ordinary least squares. The primary OLS regression equation is

$$rvol_t^{market} = \mu_1 + \sum_{i=1} \alpha_i rvol_{t-i}^{oil} + \varepsilon_t, \quad i = 1, \dots, T, \quad (1.6)$$

where $rvol^{market}$ and $rvol^{Oil}$ are our measures of market return volatility and oil price volatility, respectively. μ_1 is a constant, and ε_t is the usual error term.

1.4 Empirical Results

1.4.1 In-sample predictions

Table 1.6 contains the in-sample prediction results of equation (1.6) for the S&P 500 and CRSP stock markets and for West Texas Intermediate using realized and conditional return volatility series. Based on the results, realized price volatility of West Texas Intermediate does not significantly predict market return volatility in both S&P 500 and CRSP aggregate markets at 5 percent level during the period 1986:1-2012:12. Conditional price volatility of West Texas Intermediate also shows no significant predictive power for stock return volatility in both aggregate markets during the period 1973:10-2012:12. The positive estimated coefficient α_1 implies that an increase in oil price volatility this month causes higher stock return volatility next month, but this effect is not statistically significant. We also fail to reject the hypothesis that the West Texas Intermediate price volatility does not Granger cause stock return volatility. In order to test how robust these results are in relation to the correlation of current stock return volatility and its prior values, we run regressions in which in addition to lags of oil price volatility, we include lags of stock return volatility (regression equation (1.7)). Also, we jointly include the current oil price volatility and the lags of stock return volatility in addition to lags of oil price volatility in a separate regression (regression equation (1.8)) to see if it influences the results.

$$rvol_t^{market} = \mu_1 + \sum_{i=1} \alpha_i rvol_{t-i}^{oil} + \sum_{i=1} \beta_i rvol_{t-i}^{market} + \varepsilon_t. \quad (1.7)$$

$$rvol_t^{market} = \mu_1 + \alpha_0 rvol_t^{oil} + \sum_{i=1} \alpha_i rvol_{t-i}^{oil} + \sum_{i=1} \beta_i rvol_{t-i}^{market} + \varepsilon_t. \quad (1.8)$$

In Table 1.7 we report the estimates of equations (1.7) and (1.8). To facilitate comparison

Table 1.6: In-sample estimation results of equation (1.6) for S&P 500 and CRSP aggregate markets and for West Texas Intermediate. * and ** indicate significant coefficients at 5% and 1% level, respectively based on Newey West standard errors. BIC values indicate that the lag length is equal to one. For Granger causality test, p -values are reported, lag lengths are determined based on BIC and H_0 is: oil price volatility does not Granger cause market return volatility.

Market	1986:1-2012:12			1973:10-2012:12		
	Realized volatility			Conditional volatility		
	α_1	R^2	Granger causality	α_1	R^2	Granger causality
S&P 500	0.14	0.06	0.533	0.17	0.12	0.417
CRSP	0.14	0.07	0.602	0.13	0.08	0.226

with our previous results, the results of equation (1.5) are also included. To confirm that the evidence of predictability cannot be attributed to a specific time period, we report results for the sample period of [Driesprong et al. \(2008\)](#), which is the subperiod ending in April 2003. Furthermore, consistent with realized return volatility series, the results for conditional return volatility are also reported over the subperiods start January 1986. For equation (1.6), with the exception of the results over the period 1973:10-2012:12 that are discussed earlier in Table 1.6, we find no statistically significant evidence of predictability of S&P 500 return volatility in any of the investigated subperiods. However, when using the CRSP conditional return volatility series, the estimated coefficient is significant at a five percent level over the period 1986:1-2012:12. Over all of the investigated subperiods, Granger causality test results do not indicate that West Texas Intermediate price volatility has predictive power for stock return volatility. The predictability shown in the results of equation (1.6) for CRSP volatility over the period 1986:1-2012:12 is not robust to the inclusion of lagged stock return volatility. The results of equation (1.7) show that stock return volatility is mainly affected by itself lagged one period, and West Texas Intermediate price volatility has no significant predictive power for market return volatility.

The results from estimating equation (1.7) demonstrate that the inability of oil price volatility to predict stock return volatility is robust to the presence of contemporaneous oil price volatility. For conditional volatility, the estimation results for equation (1.8) indicates neither current oil price volatility nor one-month lagged oil price volatility significantly in-

fluenced current stock return volatility in any of the investigated time periods. For realized volatility, however, there is strong evidence of an effect of contemporaneous oil price volatility. The coefficient on current West Texas Intermediate price volatility is significantly different from zero for both aggregate markets and over both of the time periods 1986:1-2012-12 and 1986:1-2003-4.

Our in-sample prediction results provide no support for the hypothesis that oil price volatility leads stock return volatility, but there is evidence of a significant contemporaneous interaction between stock return volatility and oil price volatility after 1986. We also checked the robustness of our results to the use of real stock return volatility and excess stock return volatility rather than total returns. Real stock returns and excess stock returns were calculated by subtracting the inflation rate and one-month Treasury bill rate from stock returns, respectively. Inflation rates were calculated using the U.S. CPI. The results (not reported) showed that in both cases, the parameter estimates and related p -values were only marginally different from our reported results.

Table 1.7: In-sample estimation results of equation (1.6) compared with the results of equations (1.7) and (1.8) for S&P 500 and CRSP aggregate markets and for West Texas Intermediate. The results of equation (1.6) are based on Newey West standard errors. The results of equations (1.7) and (1.8) are based on White standard errors. * and ** indicate significant coefficients at 5% and 1% level, respectively. BIC values indicate that the lag length is equal to one in all of these regression equations. *Adj. R*²s are reported in parentheses. For Granger causality test, *p*-values are reported, lag lengths are determined based on BIC and H_0 is: oil price volatility does not Granger cause market return volatility.

Realized volatility 1986:1-2012:12							
Market	Eq. (1.6)	Eq. (1.7)		Eq. (1.8)			Granger causality
	α_1	α_1	β_1	α_0	α_1	β_1	
S&P 500	0.14 (0.06)	0.03 (0.39)	0.60**	0.12** (0.42)	-0.03	0.58**	0.533
CRSP	0.14 (0.07)	0.02 (0.42)	0.63**	0.12** (0.46)	-0.04	0.61**	0.602
Realized volatility 1986:1-2003:4							
S&P 500	0.02 (0.00)	-0.000 (0.19)	0.45**	0.07** (0.21)	-0.03	0.45**	0.915
CRSP	0.03 (0.00)	-0.001 (0.24)	0.49**	0.07** (0.26)	-0.04	0.50**	0.895
Conditional volatility 1973:10-2012:12							
S&P 500	0.17 (0.12)	0.01 (0.78)	0.87**	0.04 (0.78)	-0.03	0.87**	0.417
CRSP	0.13 (0.08)	0.01 (0.85)	0.91**	0.01 (0.85)	-0.003	0.91**	0.226
Conditional volatility 1973:10-2003:4							
S&P 500	0.09 (0.05)	0.01 (0.83)	0.90**	0.006 (0.82)	0.005	0.90**	0.184
CRSP	0.04 (0.03)	0.006 (0.87)	0.93**	-0.000 (0.87)	0.007	0.93**	0.172
Conditional volatility 1986:1-2012:12							
S&P 500	0.23 (0.14)	0.003 (0.70)	0.84**	0.15 (0.73)	-0.10	0.81**	0.878
CRSP	0.19* (0.10)	0.01 (0.87)	0.92**	0.05 (0.87)	-0.02	0.92**	0.317
Conditional volatility 1986:1-2003:4							
S&P 500	0.05 (0.01)	-0.002 (0.80)	0.90**	0.05 (0.81)	-0.03	0.90**	0.846
CRSP	0.04 (0.01)	-0.000 (0.91)	0.95**	0.01 (0.91)	-0.008	0.95**	0.974

As another robustness check, we estimate a GARCH(1,1) model for stock returns that allows us to include lagged oil price changes directly as an exogenous regressor in the conditional variance equation (a GARCH(1,1)-X model; see, for instance, [Engle & Patton, 2001](#)). The conditional variance equation can be written as

$$h_t = \omega + \gamma_1 \varepsilon_{t-1}^2 + \varphi_1 h_{t-1} + \phi rvar_{t-1}^{oil}, \quad (1.9)$$

where $rvar^{oil}$ is the conditional variance of oil price changes that is obtained by fitting a GARCH(1,1) model to monthly West Texas Intermediate log returns. Then, we compare the estimated coefficients of equation (1.9) with the estimated coefficients obtained from fitting a univariate GARCH(1,1) model to stock returns.

Table 1.8 contains the estimated coefficients of the GARCH(1,1)-X and GARCH(1,1) variance equations. The results show that the lagged variance of West Texas Intermediate does not affect the variance of stock returns as γ_1 and φ_1 are not affected and ϕ is close to zero and insignificant over all of the investigated time periods. This implies that oil price volatility does not have a significant predictive power for stock return volatility. These results are consistent with our in-sample prediction findings.

In Table 1.7, the results based on the conditional return volatility series show that β_1 is close to one in both equations (1.7) and (1.8) over all of the subsamples. Furthermore, in Table 1.8, $\gamma + \varphi_1$ is close to one in all of the subsamples. This implies that the persistence in the conditional volatility of aggregate stock returns is very high - close to a unit root. As discussed earlier, the evidence shows that our conditional return volatility series are strictly stationary based on the ADF and PP tests shown in Tables 1.2 and 1.4. The tests indicate that there is no unit root across conditional West Texas Intermediate price volatility and aggregate stock (S&P500 and CRSP) return volatility. Nonetheless, the regression models still indicate a possible problem. The possibility of a unit root in the volatility of asset returns has been studied by numerous authors, leading to the integrated GARCH (IGARCH) model,

Table 1.8: GARCH(1,1)-X and GARCH(1,1) estimation results for S&P 500 and CRSP aggregate markets and for West Texas Intermediate. * and ** indicate significant coefficients based on robust standard errors at 5% and 1% level, respectively.

Market	1973:10-2012:12							
	GARCH(1,1)-X				GARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.0001	0.1795*	0.7537**	0.0000	0.0001	0.1795*	0.7537**	
CRSP	0.0001	0.1030*	0.8531**	0.0000	0.0001	0.1030*	0.8531**	
Market	1973:10-2003:4							
	GARCH(1,1)-X				GARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00009	0.1273**	0.8104**	0.0000	0.00009	0.1273**	0.8104**	
CRSP	0.0001	0.0459	0.8983**	0.0000	0.0001	0.0459	0.8983**	
Market	1986:1-2012:12							
	GARCH(1,1)-X				GARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00013	0.2847	0.6627**	0.0000	0.00013	0.2847	0.6627**	
CRSP	0.00005	0.1295*	0.8571**	0.0000	0.00005	0.1295*	0.8571**	
Market	1986:1-2003:4							
	GARCH(1,1)-X				GARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00006	0.1950*	0.7800**	0.0000	0.00006	0.1950*	0.7800**	
CRSP	0.00004	0.0559	0.9266**	0.0000	0.00004	0.0559	0.9266**	

reviewed in [Bollerslev *et al.* \(1994\)](#). The IGARCH model introduced by [Engle & Bollerslev \(1986\)](#) is a restricted version of the GARCH model shown in equation (1.2), where the coefficients sum up to one ($\sum_{i=1}^q \gamma_i + \sum_{j=1}^p \varphi_j = 1$), implying the existence of permanent shocks to the conditional variance. However, as discussed above, IGARCH models with $\omega > 0$ are strictly stationary.

In order to test for a unit root, we first fit an IGARCH(1,1) model to West Texas Intermediate and the aggregate stock return series to see if the estimated ω is greater than zero. The results shown in Table 1.9 reveal that ω is not equal to zero, though it is very small, over all of the subsamples. Hence, the IGARCH(1,1) conditional volatility series are strictly stationary. In Table 1.9 we also report the results of equation (1.9) using an IGARCH (1,1)-X model, where the exogenous regressor is the lagged oil price conditional variance constructed from fitting an IGARCH(1,1) model to West Texas Intermediate monthly returns. The results show that ϕ is zero and insignificant over all of the investigated time periods. Also, the

Table 1.9: IGARCH(1,1)-X and IGARCH(1,1) estimation results for S&P 500 and CRSP aggregate markets and for West Texas Intermediate. * and ** indicate significant coefficients based on robust standard errors at 5% and 1% level, respectively. p -value of φ_1 is not obtained.

Market	1973:10-2012:12							
	IGARCH(1,1)-X				IGARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00005	0.2110**	0.7889	0.0000	0.00005	0.2110**	0.7889	
CRSP	0.00004	0.1371**	0.8628	0.0000	0.00004	0.1371**	0.8628	
Market	1973:10-2003:4							
	IGARCH(1,1)-X				IGARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00003	0.1539**	0.8460	0.0000	0.00003	0.1539**	0.8460	
CRSP	0.00002	0.0810**	0.9189	0.0000	0.00002	0.0810**	0.9189	
Market	1986:1-2012:12							
	IGARCH(1,1)-X				IGARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00008	0.2951	0.7048	0.0000	0.00008	0.2951	0.7048	
CRSP	0.00004	0.1429**	0.8570	0.0000	0.00004	0.1429**	0.8570	
Market	1986:1-2003:4							
	IGARCH(1,1)-X				IGARCH(1,1)			
	ω	γ_1	φ_1	ϕ	ω	γ_1	φ_1	
S&P 500	0.00004	0.2053*	0.7946	0.0000	0.00004	0.2053*	0.7946	
CRSP	0.00002	0.0759**	0.9240	0.0000	0.00002	0.0759**	0.9240	

estimated γ_1 and φ_1 are not different from their corresponding values in the IGARCH(1,1) model, and only slightly different from the values reported in Table 1.8. In addition, we reestimated equation (1.8) using the IGARCH(1,1) strictly stationary return volatility series. The results are reported in Table 1.10. The results show that in all of the estimated time periods β_1 is close to one in both aggregate markets. The findings of these tests are fully consistent with our previous findings that oil price volatility does not have significant predictive power for stock return volatility, and imply that there is not a unit root problem in the results reported in Tables 1.7 and 1.8.

As an additional check, we reestimated equation (1.8) with two, three, and four lags. The results (not reported) showed that β_1 is still close to one. There is again no evidence that oil price volatility has significant predictive power for aggregate stock return volatility.

Repeating the estimation of equation (1.8) with one lag using the first differenced return volatility series (rather than the levels of the return volatility series) does not change the

Table 1.10: In-sample estimation results of equation (1.8) using an IGARCH(1,1) model to construct conditional return volatility series. The results are reported based on White standard errors. * and ** indicate significant coefficients at 5% and 1% level, respectively. Lag lengths are determined based on BIC.

Market	1973:10-2012:12		
	α_0	α_1	β_1
S&P 500	0.04	-0.02	0.89**
CRSP	0.01	-0.02	0.92**
1973:10-2003:4			
S&P 500	0.006	0.003	0.92**
CRSP	-0.000	0.009	0.94**
1986:1-2012:12			
S&P 500	0.11	-0.09	0.84**
CRSP	0.04	-0.02	0.92**
1986:1-2003:4			
S&P 500	0.04	-0.03	0.91**
CRSP	0.01	-0.008	0.95**

finding either. Even if there is a unit root in the volatility series, it has no effect on the results shown in Table 1.7.

Our in-sample prediction results may raise the question “what about other oil price series and individual industry sectors?” To answer this question we reestimate equation (1.8) using the U.S producer price index for crude oil, the refiners’ acquisition cost for imported crude oil, and industry level stock data of fifteen oil sensitive sectors including Petroleum and Natural Gas, Automobiles and Trucks, Retail, Precious Metals, Chemicals, Rubber and Plastic Products, Construction, Steel works, Machinery, Aircraft, Shipbuilding and Railroad Equipment, Coal, Utilities, Banking, and Insurance. Since daily data for both the U.S producer price index for crude oil and the refiners’ acquisition cost for imported crude oil are not available, only conditional volatility series are used for this analysis. The results are shown in Tables 1.11 and 1.12.

Table 1.11: In-sample estimation results of equation (1.8) for S&P 500, CRSP and fifteen individual oil sensitive sectors and for WTI, PPI and RAC. The results are reported based on White standard errors. * and ** indicate significant coefficients at 5% and 1% level, respectively. Lag lengths are determined based on BIC.

Market	1974:1-2012:12								
	West Texas Intermediate			Producer Price Index			Refiners' Acquisition Cost		
	α_0	α_1	β_1	α_0	α_1	β_1	α_0	α_1	β_1
S&P 500	0.07	-0.06	0.87**	0.08	-0.07	0.86**	0.04	-0.03	0.86**
CRSP	0.03	-0.02	0.91**	0.04	-0.03	0.91**	0.01	-0.01	0.91**
Petroleum	0.02	-0.01	0.91**	-0.000	0.000	0.91**	0.003	-0.003	0.91**
Automobiles	0.08	-0.04	0.88**	0.10	-0.07	0.87**	0.05	-0.02	0.88**
Retail	0.02	-0.01	0.82**	0.03	-0.02	0.82**	0.01	-0.007	0.82**
Precious Metals	0.06	-0.04	0.85**	0.05	-0.04	0.85**	0.03	-0.02	0.85**
Chemicals	0.04*	-0.02	0.90**	0.04*	-0.03*	0.90**	0.02**	-0.02**	0.90**
Rubber	0.01	0.002	0.89**	0.02	-0.01	0.89**	0.01	-0.002	0.90**
Construction	0.02	-0.01	0.76**	0.04	-0.03	0.76**	0.01	-0.008	0.76**
Steel works	0.02	0.003	0.92**	0.02	-0.004	0.92**	0.02	-0.005	0.92**
Machinery	0.05	-0.03	0.77**	0.06	-0.04	0.76**	0.03	-0.02	0.77**
Aircraft	0.02	-0.02	0.90**	0.03	-0.03	0.90**	0.02	-0.02	0.90**
Shipbuilding	0.004	0.008	0.92**	0.01	-0.000	0.92**	0.003	0.005	0.92**
Coal	0.05*	-0.03	0.97**	0.05*	-0.04*	0.97**	0.03*	-0.02*	0.97**
Utilities	0.008	0.001	0.91**	0.01	-0.009	0.91**	0.009	-0.002	0.90**
Banking	0.02	0.01	0.89**	0.04	-0.02	0.89**	0.01	0.01	0.89**
Insurance	0.08	-0.04	0.78**	0.09	-0.07	0.78**	0.03	-0.002	0.78**
Market	1974:1-2003:4								
	α_0	α_1	β_1	α_0	α_1	β_1	α_0	α_1	β_1
	α_0	α_1	β_1	α_0	α_1	β_1	α_0	α_1	β_1
S&P 500	0.01	-0.01	0.90**	0.03	-0.03	0.90**	0.01	-0.01	0.90**
CRSP	0.004	-0.003	0.92**	0.01	-0.01	0.92**	0.002	-0.003	0.92**
Petroleum	0.02	-0.01	0.93**	-0.002	0.002	0.92**	0.002	-0.004	0.92**
Automobiles	0.01	-0.002	0.93**	0.02	-0.01	0.93**	0.009	-0.003	0.93**
Retail	0.004	0.002	0.65**	0.01	-0.008	0.65**	0.002	0.000	0.65**
Precious Metals	-0.000	0.006	0.87**	-0.006	0.01	0.87**	-0.003	-0.000	0.87**
Chemicals	0.006	-0.005	0.92**	0.009	-0.008	0.92**	0.005	-0.004**	0.92**
Rubber	0.003	-0.001	0.92**	0.008	-0.006	0.92**	0.003	-0.002	0.92**
Construction	0.002	-0.000	0.88**	0.01	-0.01	0.88**	0.003	-0.002	0.88**
Steel works	-0.007	0.01	0.96**	-0.000	0.01	0.95**	0.003	0.000	0.96**
Machinery	0.01	-0.005	0.86**	0.01	-0.01	0.85**	0.007	-0.005	0.85**
Aircraft	0.008	-0.01	0.90**	0.02	-0.03	0.90**	0.01	-0.01	0.90**
Shipbuilding	0.000	0.000	0.95**	0.000	0.000	0.95**	0.000	0.000	0.95**
Coal	0.02	-0.02	0.98**	0.03	-0.03	0.97**	0.01	-0.01	0.97**
Utilities	0.003	0.003	0.94**	0.01	-0.01	0.94**	0.006	-0.000	0.93**
Banking	0.01	-0.005	0.75**	0.03	-0.02	0.74**	0.005	-0.000	0.75**
Insurance	0.02	-0.001	0.64**	0.04	-0.03	0.64**	-0.000	0.01	0.64**

Table 1.12: In-sample estimation results of equation (1.8) for S&P 500, CRSP and fifteen individual oil sensitive sectors and for WTI, PPI and RAC. The results are reported based on White standard errors. * and ** indicate significant coefficients at 5% and 1% level, respectively. Lag lengths are determined based on BIC.

Market	1986:1-2012:12								
	West Texas Intermediate			Producer Price Index			Refiners' Acquisition Cost		
	α_0	α_1	β_1	α_0	α_1	β_1	α_0	α_1	β_1
S&P 500	0.15	-0.10	0.81**	0.18*	-0.15*	0.81**	0.13	-0.08	0.80**
CRSP	0.05	-0.02	0.92**	0.07*	-0.05	0.92**	0.04	-0.02	0.92**
Petroleum	0.01	-0.01	0.95**	0.006	-0.002	0.95**	0.008	-0.005	0.95**
Automobiles	0.14	-0.07	0.85**	0.19	-0.13	0.85**	0.14	-0.07	0.85**
Retail	0.10	-0.03	0.60**	0.14	-0.07	0.60**	0.10	-0.03	0.60**
Precious Metals	0.14	-0.08	0.20	0.15	-0.07	0.19	0.15	-0.08	0.19
Chemicals	0.06*	-0.03	0.92**	0.07**	-0.05*	0.92**	0.06**	-0.03**	0.93**
Rubber	0.02	0.007	0.92**	0.05	-0.01	0.92**	0.03	-0.002	0.93**
Construction	0.08	-0.01	0.06	0.11*	-0.05	0.05	0.07	-0.01	0.05
Steel works	0.04	0.000	0.92**	0.04	-0.01	0.92**	0.05	-0.01	0.92**
Machinery	0.10	-0.05	0.78**	0.12	-0.08	0.77**	0.10	-0.05	0.77**
Aircraft	0.06	-0.04	0.88**	0.06	-0.06	0.89**	0.06	-0.05	0.88**
Shipbuilding	0.004	0.01	0.93**	0.02	0.007	0.93**	0.01	0.009	0.93**
Coal	0.05	-0.04	0.98**	0.08*	-0.07*	0.98**	0.06*	-0.04*	0.98**
Utilities	0.01	0.001	0.90**	0.03	-0.01	0.90**	0.02	-0.01	0.90**
Banking	0.03	0.04	0.90**	0.09*	-0.01	0.89**	0.03	0.03	0.90**
Insurance	0.16	-0.02	0.71**	0.22	-0.12	0.71**	0.13	-0.009	0.71**
Market	1986:1-2003:4								
	α_0	α_1	β_1	α_0	α_1	β_1	α_0	α_1	β_1
	α_0	α_1	β_1	α_0	α_1	β_1	α_0	α_1	β_1
S&P 500	0.05	-0.03	0.90**	0.10	-0.09	0.90**	0.06	-0.04	0.89**
CRSP	0.01	-0.008	0.95**	0.02	-0.02	0.95**	0.01	-0.01	0.95**
Petroleum	0.02	-0.01	0.94**	0.008	-0.001	0.94**	0.01	-0.01	0.94**
Automobiles	0.02	-0.009	0.95**	0.04	-0.03	0.95**	0.02	-0.01	0.95**
Retail	0.04	-0.000	0.43**	0.09	-0.05	0.43**	0.07	-0.02	0.44**
Precious Metals	-0.002	0.001	0.18	0.007	0.04	0.18	0.02	-0.02	0.18
Chemicals	0.01*	-0.006	0.95**	0.01*	-0.01	0.95**	0.01*	-0.01**	0.95**
Rubber	0.01	-0.005	0.95**	0.02*	-0.01*	0.95**	0.01	-0.01	0.95**
Construction	0.002	-0.000	0.92**	0.008**	-0.005*	0.92**	0.004*	-0.001	0.92**
Steel works	-0.001	0.01	0.97**	0.01	0.003	0.97**	0.01	-0.009	0.97**
Machinery	0.02	-0.01	0.88**	0.05	-0.04	0.88**	0.03	-0.02	0.88**
Aircraft	0.04	-0.05	0.87**	0.09	-0.11	0.88**	0.07	-0.08	0.88**
Shipbuilding	0.000	-0.000	0.83**	0.000	-0.000	0.83**	0.000	-0.000	0.83**
Coal	0.02	-0.02	0.99**	0.06	-0.05	0.99**	0.03	-0.03	0.99**
Utilities	0.009	0.001	0.97**	0.03*	-0.02	0.97**	0.02*	-0.01	0.97**
Banking	0.03	0.001	0.34**	0.10*	-0.05	0.32**	0.04	-0.01	0.34**
Insurance	0.06	0.02	0.54**	0.21	-0.13	0.54**	0.09	0.009	0.55**

The results in Tables 1.11 and 1.12 are largely consistent with our previous findings for the S&P 500 and the CRSP aggregate markets using West Texas Intermediate volatility. The exception is for the 1986:1-2012:12 subsample, for which we find that the volatility of the U.S. producer price index for crude oil has predictive power for S&P 500 return volatility. However, the predictability is limited to that time period.

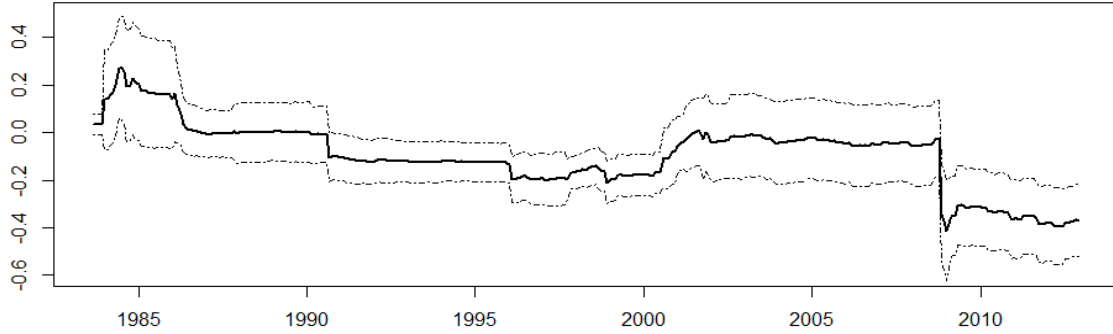
In individual industry sectors, West Texas Intermediate price volatility shows no predictive power for stock return volatility at the industry level. The refiners' acquisition cost for imported crude oil has predictive power for the chemicals industry, but this predictability is shown by the U.S. producer price index for crude oil only for the periods ending in December 2012. Furthermore, in the coal industry, both PPI and RAC show predictive power only for the periods ending in December 2012. There is also some evidence that volatility of the PPI can predict stock return volatility in the rubber and construction industries over the period 1986:1-2003:4, but that does not carry over to the other oil series or time periods. For the other industries, there is no evidence of predictability. We did the same unit root robustness checks for the individual industry sector analysis as for the aggregate markets, and it does not change our results.

1.4.2 Instability of forecasting relationships

In this section, we document the instability of the forecasting relationship between aggregate stock return volatility and oil price volatility. Figure 1.4 shows the estimation results for the forecasting regression equation (1.10) below (equation (1.8) with lag length equal to one) using 10-year rolling windows.

$$rvol_t^{market} = \mu_1 + \alpha_0 rvol_t^{oil} + \alpha_1 rvol_{t-1}^{oil} + \beta_1 rvol_{t-1}^{market} + \varepsilon_t. \quad (1.10)$$

S&P 500: 10-year rolling window estimates for one-month lagged West Texas Intermediate price volatility coefficient



CRSP: 10-year rolling window estimates for one-month lagged West Texas Intermediate price volatility coefficient

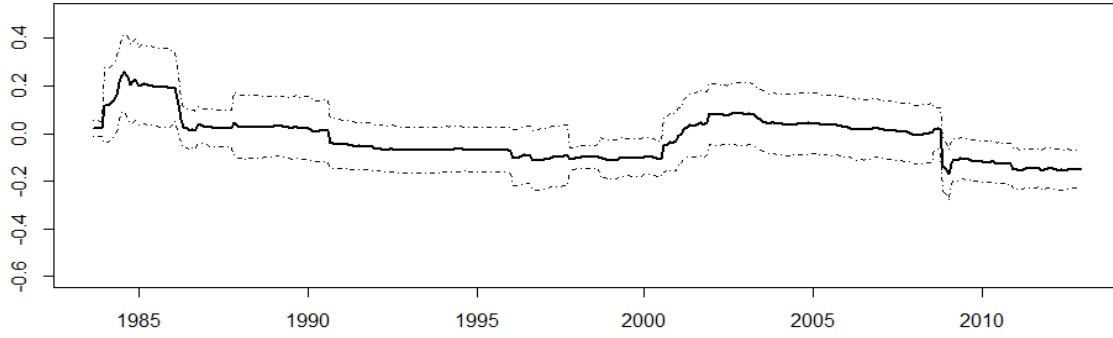


Figure 1.4: 10-year rolling window estimation results for α_1 in equation (1.10) for S&P 500 and CRSP and West Texas Intermediate over the period 1973:10-2012:12. The dashed lines denote the point estimate confidence intervals at 5% level.

The results are reported for the S&P 500 and CRSP aggregate markets and for West Texas Intermediate from 1973:10 to 2012:12. The panels of Figure 1.4 plot the estimates of the slope coefficient α_1 . The dashed lines denote the 95% confidence intervals on the point estimates.

The considerable instability of the forecasting relationship is illustrated by the variation of α_1 over time. For the S&P 500, the estimate of α_1 is close to zero in the subsamples ending in late 1983, but for the subsamples ending the mid-1980s it is 0.2. In contrast, α_1 is much smaller for the subsamples ending in the late 1990s, at around -0.2. It rises again to around zero in the subsamples ending between 2003 and 2008, and then drops to around -0.4 in the subsamples ending in the late 2000s and early 2010s. Compared to the S&P 500, the fluctuations in the estimates of α_1 for CRSP returns are smaller. The estimate of α_1

is close to zero in the subsamples ending in late 1983, it increases to 0.2 in the subsamples ending the mid-1980s, and then is much smaller (around -0.1) for the samples ending in the mid-1990s and late 1990s. It rises to around zero in the subsamples ending between 2003 and 2008, and then falls to around -0.2 in the subsamples ending in the late 2000s and early 2010s. For both the S&P 500 and CRSP aggregate markets, α_1 is statistically insignificant in all of the subsamples ending between 1983:9 and 2012:12. This evidence confirms our previous finding that oil price volatility does not forecast stock return volatility, or at least not any robust fashion. Not only do the estimated coefficients change, they even change sign within the sample.

1.4.3 Impulse response function

An alternative way to investigate the short run and long run impacts of oil price volatility shocks on stock return volatility is to use a structural vector autoregression (SVAR) model that relates U.S. aggregate stock return volatility to oil price volatility. Specifically, we estimate a SVAR model for monthly data, with the impulse response functions identified using a recursive ordering with oil price volatility first, so that oil price volatility can affect stock return volatility contemporaneously, but stock return volatility cannot contemporaneously impact oil price volatility. The corresponding reduced form VAR model is

$$Z_t = \alpha + \sum_{i=1}^k A_i Z_{t-i} + \varepsilon_t, \quad (1.11)$$

where Z_t denotes a time series vector consisting of oil price volatility and stock return volatility, in that order, ε_t is a vector of serially and mutually uncorrelated structural innovations and lag length, k , is determined based on AIC.

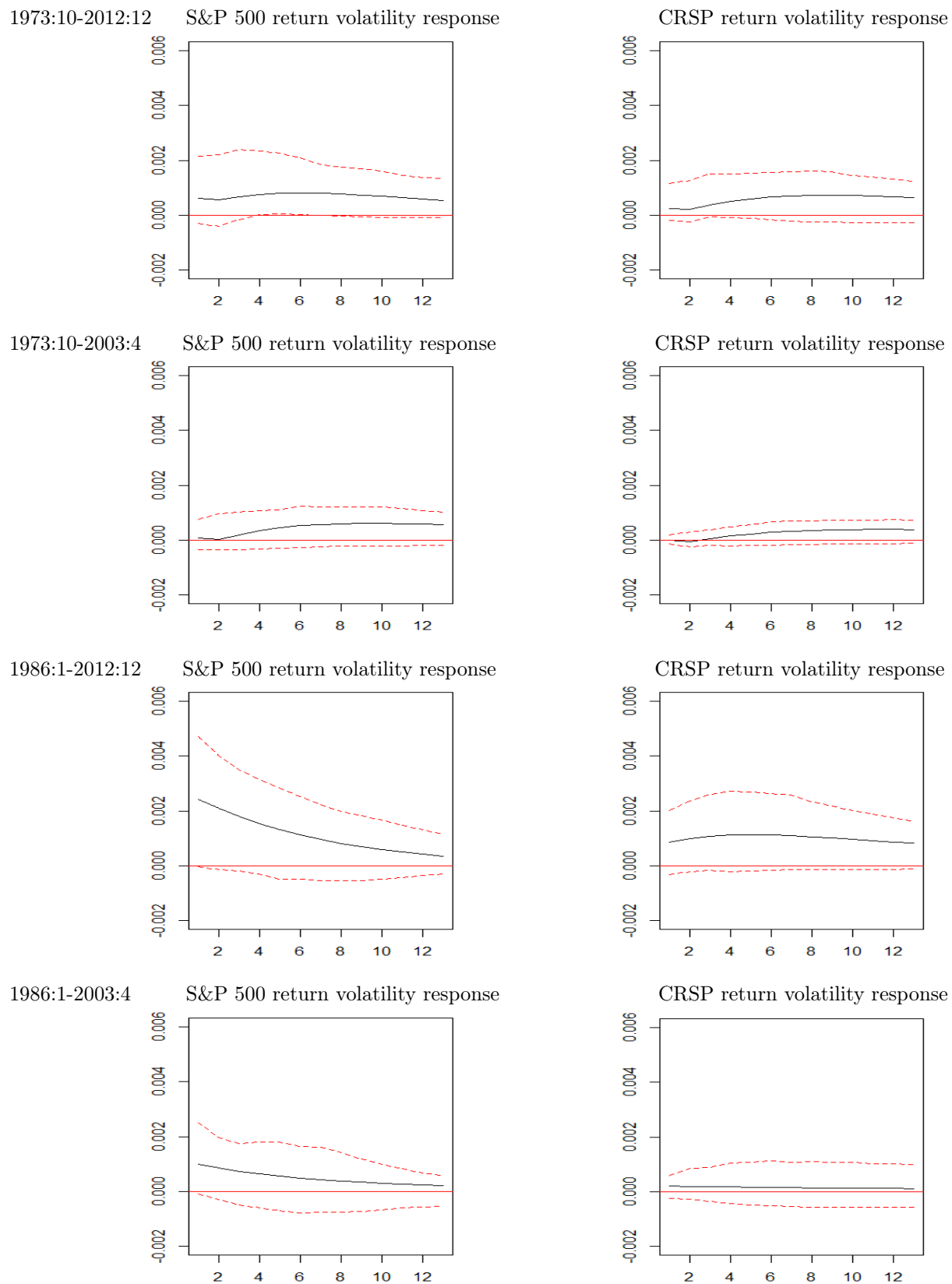


Figure 1.5: Responses of S&P 500 and CRSP return volatility to a one-standard deviation shock to West Texas Intermediate price volatility.

The reduced-form VAR residual vector, e_t , takes the form

$$e_t = \begin{pmatrix} e_{1t}^{oil\ price\ changes\ volatility} \\ e_{2t}^{stock\ returns\ volatility} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} \varepsilon_{1t}^{oil\ price\ changes\ volatility\ shock} \\ \varepsilon_{2t}^{other\ shocks\ to\ stock\ returns\ volatility} \end{pmatrix}. \quad (1.12)$$

Figure 1.5 shows the impulse responses resulting from a one-standard deviation shock to West Texas Intermediate volatility in different subsamples. Confidence intervals are computed using bootstrap methods.

The central result in Figure 1.5 is that stock return volatility does not show a response in the short run to oil price volatility shocks. During the period 1973:10-2012:12, an unexpected increase in West Texas Intermediate volatility causes a delayed and very small increase in the S&P 500 and CRSP return volatility, with the effect being more persistent for CRSP return volatility. The impulse responses follow a similar pattern for the 1973:10-2003:4 subsample. Over the period 1986:1-2012:12, a one-standard deviation shock to West Texas Intermediate volatility causes a very small, immediate increase in the S&P 500 return volatility, followed by a gradual decline. CRSP return volatility shows a very small but sustained response to that shock. During the period 1986:1-2003:4, aggregate stock markets show similar responses in a weaker form.

1.4.4 Out-of-sample predictions and forecast evaluation

In the forecasting literature, it is well-known that a model that provides the best in-sample fitting does not necessarily provide the most accurate forecast (see for example [Hendry & Clements, 2003](#) and [Hendry & Ericsson, 2003](#)). To deal with this, researchers employ out-of-sample forecast techniques in addition to in-sample estimates. In out-of-sample prediction the data is divided into two subperiods. The first subperiod is used to fit the model and is called the estimation subperiod, with the remaining data representing the forecasting subperiod, and being used to evaluate the performance of each forecasting model. In order

to assess out-of-sample forecast performance, consider regression equation (1.13) (equation (1.6) with lag length equal to one) below.

$$rvol_t^{market} = \mu_1 + \alpha_1 rvol_{t-1}^{oil} + \varepsilon_t, \quad t = 1 \dots T. \quad (1.13)$$

We first divide the total sample of T observations for $rvol^{market}$ and $rvol^{oil}$ into a estimation subsample including the first R observations and a forecasting subsample composed of the last $T - R = S$ observations. The first out-of-sample forecast of stock return volatility is given by $\hat{rvol}_{R+1}^{market} = \hat{\mu}_1 + \hat{\alpha}_1 rvol_R^{oil}$, where $\hat{\mu}_1$ and $\hat{\alpha}_1$ are the OLS estimates of μ_1 and α_1 , respectively, using the first R observations. The next out-of-sample forecast of stock return volatility is given by $\hat{rvol}_{R+2}^{market} = \hat{\mu}_1 + \hat{\alpha}_1 rvol_{R+1}^{oil}$, where $\hat{\mu}_1$ and $\hat{\alpha}_1$ are the OLS estimates of μ_1 and α_1 , respectively, using the period $t = 1 \dots R + 1$. Proceeding in this manner through the end of the forecasting subperiod, a series of S out-of-sample forecasts of stock return volatility can be computed using $\{rvol_t^{oil}\}_R^{T-1}$.

In order to figure out whether oil price volatility has significant out-of-sample predictive power for stock return volatility, we compare equation (1.5) with equation (1.7). The purpose of this comparison is to find out whether oil price volatility contains any information that can significantly assist with out-of-sample prediction of market return volatility. The out-of-sample forecast performance of a particular model can be sensitive to how the sample split is chosen. Since no theoretical guidance exists on how to determine the split point, in order to see whether the results are robust to the choice of sample split it is important to use different split points. After estimating equations (1.5) and (1.7) for a given subsample, we perform the test that [Mincer & Zarnowitz \(1969\)](#) introduced along with two additional pairwise comparison tests that are commonly used in the literature to evaluate which equation provides significantly more accurate forecast.

The idea of Mincer-Zarnowitz test is to test the unbiasedness and efficiency of the forecast. In order to perform this test, we regress the observed values of stock return volatility on their

forecasted values over the different subperiods given in equation (1.14) below.

$$rvol_t^{market} = \theta_0 + \theta_1 \hat{rvol}_t^{market} + \varepsilon_t. \quad (1.14)$$

Then, we jointly test the hull hypothesis of $\theta_0 = 0$ and $\theta_1 = 1$. An intercept of zero indicates that the forecast is unbiased. A slope of one indicates that the forecasted market return volatility efficiently explains its observed values.

The second statistic we report is the out-of-sample mean squared error of each model over each of the forecasting subperiods. Out-of-sample mean squared error is defined as

$$MSE = \frac{1}{S} \sum_{R+1}^T \left(rvol_t^{market} - \hat{rvol}_t^{market} \right)^2, \quad (1.15)$$

where the first R observations compose the estimation subperiod, and the last S observations are considered as the forecasting subperiod. The model with lower MSE provided better forecasts over the forecasting period.

It is possible that one model can have a lower MSE over any given subsample even if both models predict equally well. Indeed, it will always be the case that the two models will have different MSE statistics in finite samples, even if the two models have the same expected loss. That is the motivation for the predictive ability test introduced by [Diebold & Mariano \(1995\)](#). We calculate the loss differential series as

$$d_t = e_{t,(1.5)}^2 - e_{t,(1.7)}^2, \quad (1.16)$$

where $e_{t,(1.5)}$ and $e_{t,(1.7)}$ are the forecast errors of equations (1.5) and (1.7), respectively, in time t . Under the null hypothesis that the two models have equal expected loss, the mean of the loss differential series is zero. [Diebold & Mariano \(1995\)](#) and [West \(1996\)](#) show the conditions under which the distribution of d is normal. The test is implemented by regressing the loss differential series on a constant and testing if the estimated coefficient is equal to

zero. As there is no restriction that the residual of that regression will be homoskedastic or serially uncorrelated, we use Newey-West standard errors. If the estimated coefficient is positive and significant, then equation (1.7) provides significantly better forecasts than equation (1.5), while a negative and significant coefficient implies that equation (1.5) provides more accurate forecasts than equation (1.7).

Table 1.13: Out-of-sample forecast evaluation results to compare the predictive abilities of equations (1.5) and (1.7). For each time period, we use the first two thirds of the sample as the estimation subperiod and the last third of the sample as the forecasting subperiod. For the periods 1973:10-2012:12, the estimation subperiod is 1973:10-1999:11. For the periods 1973:10-2003:4, the estimation subperiod is 1973:10-1993:6. For the periods 1986:1-2012:12, the estimation subperiod is 1986:1-2003:12. For the periods 1986:1-2003:4, the estimation subperiod is 1986:1-1997:7. For Mincer-Zarnowitz test, p -values are reported. * and ** indicate significance at 5%, and 1%, respectively.

Market	Realized volatility 1986:1-2012:12				
	Mincer-Zarnowitz test		MSE		Diebold-Mariano test
	Eq. (1.5)	Eq. (1.7)	Eq. (1.5)	Eq. (1.7)	t -value
S&P 500	0.017*	0.027*	2.71E-5	2.69E-5	0.94
CRSP	0.053	0.067	2.56E-5	2.55E-5	0.47
S&P 500	Realized volatility 1986:1-2003:4				
	0.000**	0.000**	2.26E-5	2.27E-5	-1.29
	0.000**	0.000 **	2.55E-5	2.56E-5	-1.34
CRSP	Conditional volatility 1973:10-2012:12				
	0.805	0.788	4.84E-5	4.84E-5	-0.43
	0.350	0.474	1.29E-5	1.28E-5	0.70
S&P 500	Conditional volatility 1973:10-2003:4				
	0.914	0.898	1.29E-5	1.30E-5	-0.47
	0.582	0.574	2.98E-6	2.98E-6	-0.29
CRSP	Conditional volatility 1986:1-2012:12				
	0.915	0.897	1.03E-4	1.03E-4	-1.88
	0.502	0.593	1.93E-5	1.91E-5	1.32
S&P 500	Conditional volatility 1986:1-2003:4				
	0.175	0.169	4.00E-5	4.01E-5	-1.29
	0.079	0.074	6.83E-6	6.86E-6	-1.56

In Table 1.13 we report the out-of-sample forecast evaluation results. The Mincer-

Zarnowitz test results indicate that there is no difference between equation (1.5) and equation (1.7) in terms of predictive ability. Both regressions provide equally accurate out-of-sample forecasts for the CRSP aggregate stock return realized volatility series over the period 1986:12-2012:12 and for both S&P 500 and CRSP aggregate markets for conditional return volatility series over all of the investigated time periods. The MSE comparisons reveal that both equations (1.5) and (1.7) forecast stock return volatility with approximately the same level of accuracy. The Diebold-Mariano test results provide no evidence against the null hypothesis of equal forecast accuracy. The findings show that West Texas Intermediate volatility does not have significant out-of-sample predictive power for S&P 500 and CRSP return volatility. These out-of-sample prediction results are in line with our previous findings.

We checked the robustness of these results to changes in the size of the estimation and forecasting subperiods. The results (not reported) were consistent with the reported out-of-sample findings and indicate that equation (1.5) forecasts stock return volatility as accurately as equation (1.7), and in some periods even provides better predictions.

1.5 Conclusions

This chapter has provided a detailed analysis of the relationship between stock return volatility and oil price volatility. We studied volatility of aggregate stock markets as well as individual industry sectors. The most important motivation for our analysis is that of predictability: if stock returns can be predicted by lagged oil price changes, as some papers in the literature have found, it may be the case that stock return volatility is predictable using lagged oil price volatility. In addition, we looked at how the relationship has changed through time. We did a variety of robustness checks to confirm that our findings are not an artifact of a particular choice of model specification.

The in-sample prediction results indicate that the stock return volatility is mainly explained by its own past and contemporaneous oil price volatility. We find little evidence to

support the view that oil price volatility leads aggregate stock return volatility. In addition, rolling window estimation results find substantial instability of the forecasting relationship through time. There are even changes in the sign of the relationship.

The results are also consistent with the results obtained from a GARCH(1,1)-X which allows for including the lagged variance of oil price changes as an exogenous regressor in the stock returns variance equation. For individual industries, we find some evidence of predictability for four of fifteen sectors including chemicals, coal, rubber and construction industries, but that finding is only robust and stable for the chemicals industry.

We then employed two alternative methodologies. First, estimated impulse response functions indicate that shocks to oil price volatility have no systematic relationship with future movements in stock return volatility. Second, an out-of-sample forecast analysis support the in-sample findings. Overall, we conclude that there is little evidence that oil price volatility can be used to predict stock return volatility.

Chapter 2

Non-Exclusionary Input Prices¹

2.1 Introduction

In traditional infrastructure industries, including telecommunications, electric power and natural gas, it is common for an upstream monopolist to supply an input that is essential for downstream production. For example, in telecommunications markets, the competition may require that rivals be able to access to the local distribution network for the origination and termination of wireless/wireline calls. In electric power and natural gas industries, the essential input takes the form of access to the transmission and distribution networks of regional power and gas companies.

In the recent literature in regulatory economics, the principal focus is on essential input (access) pricing to protect against the foreclosure of efficient downstream competition. In the case of foreclosure, a vertically-integrated provider (VIP) that is a monopoly supplier of an essential input reduces the profit margins of its rivals in the downstream market through a price squeeze and thereby drives them from the market (see for example [Perry, 1989](#); [Kahn & Taylor, 1994](#); and [Armstrong, 2002](#), among others). The exclusionary effect of a price squeeze derives from an upstream input price that is too high relative to the downstream retail price.

¹This chapter is the extended version of a published paper (see [Nadimi & Weisman, 2014](#)).

[Hausman & Tardiff \(1995\)](#) show that the price of the VIP’s retail service should be no lower than the sum of the incremental cost of providing the retail service and the net contribution foregone (opportunity cost) in selling the retail service rather than the access service to competitors. [Weisman \(2002\)](#) studies the foreclosure problem when the price of the “essential input” is determined by a regulator, and explores how high the VIP’s retail price should be in order to prevent an anticompetitive retail price squeeze.

A complementary literature addresses a different form of market exclusion, and studies the incentives of a VIP with monopoly in input market to engage in non-price discrimination or sabotage against its rivals (see for example [Salop & Scheffman, 1983](#); [Krattenmaker & Salop, 1986](#); [Sibley & Weisman, 1998a](#); [Sibley & Weisman, 1998b](#); and [Beard *et al.*, 2001](#), among others). In the case of sabotage, the VIP degrades the quality of access to downstream competitors and thereby raises the costs of its rivals. The incentive for sabotage derives from an upstream input price that is too low relative to the downstream retail price. To understand the intuition underlying this statement, recognize that when the input price is sufficiently remunerative, the VIP would not rationally choose to restrict the demand for the input by raising its rival’s costs.

[Mandy \(2000\)](#) finds that sabotage depends on three parameters including the access charge markup, the extent of downstream competition, and the relative inefficiency of the VIP in the downstream market. He shows that some combination of these parameters would lead to sabotage, while some others would not. [Mandy & Sappington \(2007\)](#) find that while some forms of sabotage may raise the operating cost of the rivals (e.g., engaging in protracted litigation and imposing standards that are costly for rivals to adopt), other forms of sabotage may primarily decrease the demand for rivals’ products (e.g., degrading the relative quality of access provided to rivals and limiting their ability to test new products and deliver them to customers). They show that the VIP’s incentives to sabotage the activities of downstream rivals can be influenced by both the type of sabotage (cost-increasing or demand-reducing) and the nature of downstream competition.

We have three important motivations for our paper. The first motivation is to establish the connection between these two strands of the literature by exploring the relationship between input prices and market exclusion when the products are differentiated. The second motivation is to examine the role of product differentiation in circumscribing the range of non-exclusionary input prices and explore how this safe harbor range changes as products become increasingly homogeneous. The third motivation concerns the importance of the problem for the antitrust or regulatory authority. While the regulators have recognized the need to be proactive in protecting against a price squeeze through price-floor constraints, to date they have not considered the important role of price-ceiling constraints to prevent sabotage. The antitrust or regulatory authority may choose to simultaneously employ the well-known price-floor constraint and a complementary price-ceiling constraint to safeguard against both forms of market exclusion. Hence, the focus of this paper is not to perform a welfare analysis *per se*, but rather to delineate regulatory pricing rules that protect against market exclusion and the integrity of the competitive process in relation to individual competitors.

The principal findings of this analysis are as follows. First, an input price that is “too high” can give rise to inefficient foreclosure, whereas an input price that is “too low” can induce the VIP to engage in sabotage or non-price discrimination. Second, there is a range of non-exclusionary input prices that simultaneously protects against both inefficient foreclosure and sabotage. This range of non-exclusionary input prices is circumscribed by the input prices generated on the basis of upper-bound and lower-bound displacement ratios. Third, the admissible range of the ratio of downstream to upstream “price-cost” margins is increasing in the degree of product differentiation and reduces to a single ratio in the limit as the products become perfectly homogeneous.

The remainder of this paper is organized as follows. Section 2.2 introduces the notation and definitions. In Section 2.3 we develop the formal model and present the main findings. Section 2.4 provides a brief summary and concluding remarks. The proofs of all formal results are contained in the Appendix A.

2.2 Notation and Definitions

There is a single VIP that serves as a monopolist in the upstream input market and a single independent downstream provider. The downstream demand functions for the VIP and the independent provider are given by $Q^V(P^V, P^I)$ and $Q^I(P^I, P^V)$, where P^i , $i = V$ and I denote the respective downstream prices for the VIP and the independent rival. The downstream outputs of the VIP and the independent downstream provider are imperfect substitutes so that $Q_{P^j}^i > 0$ for $i, j = V$ and I , $i \neq j$, where the subscripts denote partial derivatives. There are no income effects.

The price and constant marginal cost of the input are denoted by w and c , respectively. The production technology is fixed-coefficient: each unit of downstream output requires one unit of the VIP-supplied input and one unit of a complementary input. The cost of each unit of the complementary input is denoted by s^i , $i = V$ and I . Let $d > 0$ denote the increment by which the VIP raises the per-unit cost of its rival through non-price discrimination. Finally, let $C(d)$ denote the cost of non-price discrimination for the VIP, with $C(0) = 0$, $C'(0) = 0$, $C''(d) > 0$, and $C'''(d) > 0 \forall d > 0$.

The profit functions for the VIP and the independent rival, which are assumed to satisfy standard regularity conditions that ensure a unique optimum, are given, respectively, by

$$\Pi^V = Q^I(P^I, P^V)(w - c) + Q^V(P^V, P^I)(P^V - c - s^V) - C(d) \quad (2.1)$$

and

$$\Pi^I = Q^I(P^I, P^V)(P^I - w - s^I - d). \quad (2.2)$$

Assumption 2.1. $\left| \frac{\partial Q^i}{\partial P^i} \right| > \frac{\partial Q^i}{\partial P^j}$, $i, j = V$ and I , $i \neq j$.

Assumption 2.1 imposes the standard regularity condition that own-price effects dominate

cross-price effects (Vives, 1999, p. 157).

Definition 2.1. (Displacement ratio):

The displacement ratio is the absolute value of the change in the output of the independent rival associated with a one-unit increase in the output of the VIP (Armstrong *et al.*, 1996).

In the differentiated products setting under examination, there are two downstream prices and therefore two displacement ratios.

Lemma 2.1. The upper-bound displacement ratio is given by

$$\sigma_u = \left| \frac{\partial Q^I}{\partial P^I} / \frac{\partial Q^V}{\partial P^I} \right|. \quad (2.3)$$

Lemma 2.2. The lower-bound displacement ratios given by

$$\sigma_l = \left| \left(\frac{\partial Q^I}{\partial P^V} + \frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial P^V} \right) / \frac{\partial Q^V}{\partial P^V} \right|. \quad (2.4)$$

Definition 2.2. (Product homogeneity):

The degree of product homogeneity is given by $\theta = \frac{\sigma_l}{\sigma_u} \in (0, 1)$.

Assumption 2.2. The displacement ratios, σ_i , $i = l$ and u , are constants.

The VIP is generally required by the antitrust or regulatory authority to satisfy a price floor (P-F) constraint. This constraint requires that the downstream price for the VIP be no lower than the incremental cost of providing downstream output plus the net contribution foregone (opportunity cost) in not providing the upstream input. The opportunity cost in this setting is computed on the basis of σ_l because it is the change in the VIP's price

(P^V) rather than the rival's price (P^I) that induces the change in the VIP's output. This constraint requires that the lower-bound displacement ratio be no greater than the ratio of downstream to upstream “price-cost” margins (r) .

Definition 2.3. (P-F constraint): ²

$$P^V \geq c + s^V + \sigma_l(w - c) \Leftrightarrow w \leq c + \sigma_l^{-1}(P^V - c - s^V) \Leftrightarrow r = \frac{P^V - c - s^V}{w - c} \geq \sigma_l. \quad (2.5)$$

An input price that is too low relative to the output price can give rise to non-price discrimination and underscores the need for a complementary, price-ceiling (P-C) constraint. This constraint requires that the upper-bound displacement ratio be no less than the ratio of downstream to upstream “price-cost” margins.

Definition 2.4. (P-C constraint):

$$P^V \leq c + s^V + \sigma_u(w - c) \Leftrightarrow w \geq c + \sigma_u^{-1}(P^V - c - s^V) \Leftrightarrow r = \frac{P^V - c - s^V}{w - c} \leq \sigma_u. \quad (2.6)$$

The upper-bound displacement ratio (σ_u) enters the analysis here because it is the change in the rival's price (P^I) , triggered by the non-price discrimination and the resultant increase in its costs, that diverts demand from the rival to the VIP.

A binding P-C constraint defines the lower bound input price, \underline{w} , and a binding P-F constraint defines the upper bound input price, \bar{w} .

Definition 2.5. (Lower/upper-bound input prices and margin ratios):

a) The lower-bound input price (upper-bound margin ratio) is given by

$$\underline{w}_{(\sigma_u^{-1})} = c + \sigma_u^{-1}(P^V - c - s^V) \Leftrightarrow \bar{r} = \frac{P^V - c - s^V}{w - c} = \sigma_u. \quad (2.7)$$

²The input price that results when this last relation holds with equality is a form of the efficient component pricing rule or ECPR.

b) The upper-bound input price (lower-bound margin ratio) is given by

$$\bar{w}_{(\sigma_l^{-1})} = c + \sigma_l^{-1}(P^V - c - s^V) \Leftrightarrow \underline{r} = \frac{P^V - c - s^V}{w - c} = \sigma_l. \quad (2.8)$$

2.3 Formal model

The VIP and the independent rival compete in a three-stage, Bertrand-Nash game.³ In the first stage, the regulator chooses the input pricing rule, $w = c + k(P^V - c - s^V)$, where k is the inverse displacement ratio. In the second stage, the VIP and the independent rival simultaneously choose profit-maximizing prices. In the third stage, the VIP chooses the profit-maximizing level of non-price discrimination (d).

The necessary first-order conditions for the second stage of the game are given by

$$\frac{\partial \Pi^V}{\partial P^V} = \frac{\partial Q^I}{\partial P^V}(w - c) + \frac{\partial w}{\partial P^V}Q^I(P^I, P^V) + \frac{\partial Q^V}{\partial P^V}(P^V - c - s^V) + Q^V(P^V, P^I) = 0 \quad (2.9)$$

and

$$\frac{\partial \Pi^I}{\partial P^I} = \frac{\partial Q^I}{\partial P^I}(P^I - w - s^I - d) + Q^I(P^I, P^V) = 0. \quad (2.10)$$

Lemma 2.3. At the Nash equilibrium defined by (2.9) and (2.10), $\frac{\partial P^{I*}}{\partial d} > 0$ and $\frac{\partial P^{V*}}{\partial d} > 0$.

The first proposition establishes that the VIP does not engage in non-price discrimination for any input price that is greater than or equal to the lower-bound input price.

Proposition 2.1. At the Nash equilibrium, $d^* = 0 \ \forall k \geq \sigma_u^{-1} \Rightarrow \forall w \geq \underline{w}$.

The second proposition establishes that the VIP engages in non-price discrimination for any input price that is strictly less than the lower-bound input price.

³Weisman (2014) examines a similar problem in which the VIP is the leader and the rival is the follower.

Proposition 2.2. At the Nash equilibrium, $d^* > 0 \forall k < \sigma_u^{-1} \Rightarrow \forall w < \underline{w}$.

The third proposition establishes that the VIP engages in neither type of market exclusion for input prices that satisfy both the P-F and P-C constraints.

Proposition 2.3. The VIP does not engage in market exclusion $\forall k \in [\sigma_u^{-1}, \sigma_l^{-1}] \Rightarrow \forall w \in [\underline{w}, \bar{w}]$.

As shown in Figure 2.1, market exclusion takes the form of inefficient foreclosure when $w > \bar{w}$ and sabotage when $w < \underline{w}$. The range of non-exclusionary input prices is therefore defined by $w \in [\underline{w}, \bar{w}]$.

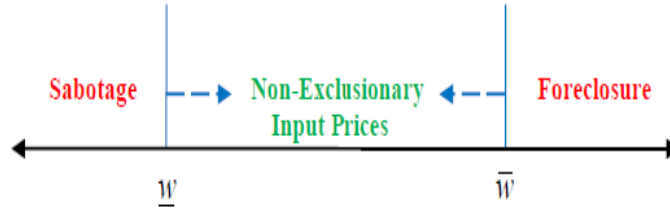


Figure 2.1: Range of non-exclusionary input prices.

The fourth proposition reveals that the range of admissible margin ratios reduces to a single ratio in the limit as the degree of product differentiation vanishes.

Proposition 2.4. In the limit as $\theta \rightarrow 1$, $\underline{r} \rightarrow \bar{r}$.

Corollary 2.1. In the limit as $\theta \rightarrow 1$, the non-exclusionary margin ratio is unique and satisfies the “equal-margin rule.”

The “equal-margin rule” requires that the input price be set so as maintain equality between the VIP’s retail and wholesale margins, or $P^V - c - s^V = w - c$.

To facilitate a closed-form solution, we specify a linear demand system of the form

$$Q^V(P^V, P^I) = a^V - b^V P^V + g^V P^I \quad (2.11)$$

and

$$Q^I(P^I, P^V) = a^I - b^I P^I + g^I P^V, \quad (2.12)$$

where $a^V, a^I, b^V, b^I, g^V, g^I > 0$.

Proposition 2.5. The Nash equilibrium prices are given by

$$P^V(k) = -\frac{2a^V + (c + d + s^I - k(s^V + c))(g^V - kb^I)}{k(3g^I + g^V - kb^I) - 4b^V + \frac{g^I g^V}{b^I}} - \frac{2(c + s^V)(b^V - kg^I) + a^I(k + \frac{g^V}{b^I})}{k(3g^I + g^V - kb^I) - 4b^V + \frac{g^I g^V}{b^I}} \quad (2.13)$$

and

$$P^I(k) = -\frac{(b^V - kg^I)((g^I(c + s^V) - b^I(ck + ks^V - 2s^I) + 2b^I(c + d) + a^I))}{g^I g^V - b^I(4b^V + k^2 b^I - k(3g^I + g^V))} - \frac{kb^I(ka^I + a^V) + a^V g^I + a^I b^V}{g^I g^V - b^I(4b^V + k^2 b^I - k(3g^I + g^V))}. \quad (2.14)$$

Proposition 2.6. The VIP's Nash equilibrium profit function is given by

$$\Pi^V(k) = \frac{\left(a^I(g^V + kb^I) + b^I(c + d + s^I)(g^V - kb^I) + 2a^V b^I \right)^2 + (c + s^V)(g^I g^V + b^I(kg^I - 2b^V))}{(g^I g^V - b^I(4b^V + k^2 b^I - k(3g^I + g^V)))^2} (b^V - kg^I) - C(d). \quad (2.15)$$

The following example illustrates the manner in which the range of non-exclusionary input prices varies with the degree of product differentiation.

Example 2.1. The demand functions for the VIP and the independent rival are linear as specified in (2.11) and (2.12) with $a^V = a^I = 20$, $b^V = b^I = 2$, and $g^V = g^I = g \in (0, 2)$. Also, $s^V = s^I = c = 1$. The numerical simulations are shown in Tables 2.1 and 2.2.⁴

The simulation results in Tables 2.1 and 2.2 confirm the theoretical findings and provide

⁴Lemmas 2.1 and 2.2 along with (2.11), (2.12) and (A.20) imply that $\sigma_u = \frac{b^I}{g^V}$ and $\sigma_l = 2b^I \left(\sqrt{(g^I)^2 + 8b^I b^V} - g^I \right)^{-1}$.

additional insights. First, P^V is increasing in θ , *ceteris paribus*. As the degree of product homogeneity increases, the VIP's upstream and downstream outputs become closer substitutes for one another which the VIP leverages by raising its retail price. Second, consistent with Proposition 2.4, the range of upper/lower-bound margin ratios ($\bar{r} - \underline{r}$) approaches zero as θ approaches unity as illustrated in Figure 2.2. Third, consistent with Corollary 2.1, in the limit as $\theta \rightarrow 1$, $P^V - c - s^V = w - c$, which is the equal margin rule. Fourth, the upper-bound input price, $\bar{w}_{(\sigma_u^{-1})}$, enables the rival to earn positive profits when the services of the VIP and the rival are sufficiently differentiated (i.e., lower values of θ) but the rival's profits approach zero as the services become increasingly homogeneous. The more homogeneous are the product offerings, the greater the ability of the VIP to appropriate the independent rival's profits through the choice of input price.⁵ Finally, the VIP's profits are increasing in θ across both inverse displacement ratios.

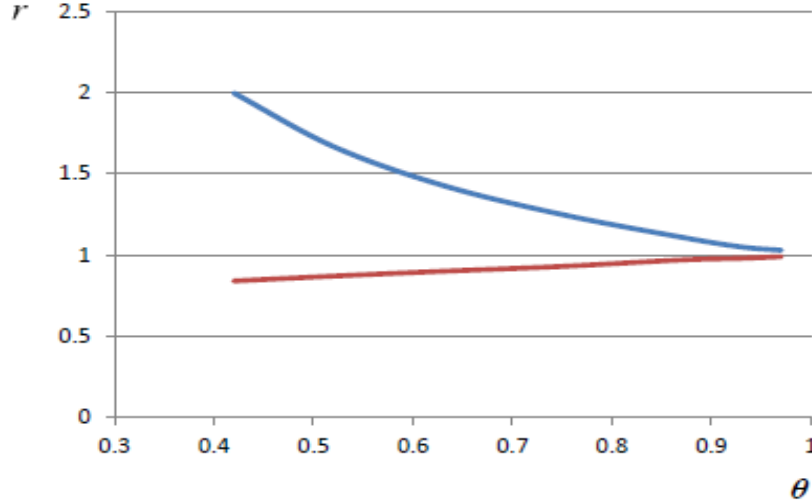


Figure 2.2: Changes in lower-bound (in red) and upper-bound (in blue) margin ratios.

⁵Note that the VIP's market power is increasing the degree of product homogeneity in this setting.

Table 2.1: Numerical simulation results.

g	σ_u^{-1}	σ_l^{-1}	θ	$\underline{w}_{(\sigma_u^{-1})}$	$\overline{w}_{(\sigma_l^{-1})}$	$P_{(\sigma_u^{-1})}^V$	$P_{(\sigma_l^{-1})}^V$	$P_{(\sigma_u^{-1})}^I$	$P_{(\sigma_l^{-1})}^I$
0.5	0.25	1.29	0.19	2.42	10.03	7.67	8.98	7.67	11.64
1.0	0.50	1.19	0.42	5.50	15.13	11.00	13.91	11.00	16.54
1.2	0.60	1.15	0.52	7.90	18.92	13.50	17.64	13.50	20.25
1.4	0.70	1.11	0.63	11.97	25.21	17.67	23.87	17.67	26.46
1.6	0.80	1.07	0.75	20.20	37.75	26.00	36.36	26.00	38.92
1.8	0.90	1.03	0.87	45.10	75.29	51.00	73.84	51.00	76.38
1.9	0.95	1.02	0.93	95.05	150.31	101.00	148.84	101.00	151.36
1.94	0.97	1.01	0.96	161.70	250.32	167.67	248.84	167.67	251.35

Table 2.2: Numerical simulation results.

g	σ_u^{-1}	σ_l^{-1}	θ	$\Pi_{(\sigma_u^{-1})}^V$	$\Pi_{(\sigma_l^{-1})}^V$	$\Pi_{(\sigma_u^{-1})}^I$	$\Pi_{(\sigma_l^{-1})}^I$	\bar{r}	\underline{r}
0.5	0.25	1.29	0.19	60.21	65.83	36.13	0.73	4.00	0.77
1.0	0.50	1.19	0.42	121.50	115.51	40.50	0.34	2.00	0.84
1.2	0.60	1.15	0.52	169.28	152.98	42.32	0.22	1.67	0.87
1.4	0.70	1.11	0.63	250.35	215.50	44.18	0.12	1.43	0.90
1.6	0.80	1.07	0.75	414.72	340.56	46.08	0.06	1.25	0.93
1.8	0.90	1.03	0.87	912.38	715.67	48.02	0.01	1.11	0.97
1.9	0.95	1.02	0.93	1911.20	1465.75	49.01	0.003	1.05	0.98
1.94	0.97	1.01	0.96	3244.05	2465.78	49.40	0.001	1.03	0.99

2.4 Conclusions

Market exclusion is a concern when input prices are “too high” because it can result in inefficient foreclosure as well as when input prices are “too low” because it can create incentives for sabotage. Upper/lower-bound displacement ratios are used to generate a range of non-exclusionary input prices. The admissible range of the ratio of downstream to upstream margins is increasing in the degree of product differentiation and reduces to a single ratio in the limit as the products become perfectly homogeneous. An important implication for competition policy is that both price-floor and price-ceiling constraints may be necessary to protect against market exclusion in certain settings.

Chapter 3

Non-Exclusionary Input Prices under Quantity Competition

3.1 Introduction

In regulated industries, including telecommunications, electric power, natural gas and railroads, it is common that a vertically integrated provider (VIP) is a monopoly supplier of an essential input for the rival to produce downstream output. Market exclusion in the form of inefficient foreclosure and sabotage can arise when input prices are, respectively, “too high” and “too low” relative to the downstream price. The first branch of the literature on market exclusion focuses on how to price the essential input so as not to foreclose efficient downstream competition (see for example [Hausman & Tardiff, 1995](#) and [Weisman, 2002](#), among others). The second branch of the literature on market exclusion explores the conditions under which the VIP does not have incentives to engage in sabotage or non-price discrimination against its rivals (see for example [Sibley & Weisman, 1998a](#); [Sibley & Weisman, 1998b](#); and [Mandy & Sappington, 2007](#), among others).

While a large body of work has been devoted to study each of these two types of market exclusion, few researchers have explored the formal connection between these two branches

of the literature. [Nadimi & Weisman \(2014\)](#) show how complementary price-floor and price-ceiling constraints can simultaneously safeguard against both forms of market exclusion in a Bertrand competition framework with a VIP that is a monopolist in the input (downstream) market. [Weisman \(2014\)](#) studies the role of complementary price-floor and price-ceiling constraints to derive a range of safe harbor input prices in a price-competition framework in which the VIP is the leader and the rival is the follower.

We have two primary objectives for this analysis. The first objective is to examine the role of product differentiation in circumscribing the range of non-exclusionary input prices in a Stackelberg quantity-competition framework in order to understand how our previous findings change under different competition settings. Second, it is reasonable to believe that the range of non-exclusionary input prices varies with the form of competition that prevails in the industry. As a result, it is important for government regulators to understand the specific model of competition that applies for the particular industry that they regulate so that the findings of the analysis can be applied in a credible and efficient manner.

The principal findings of this paper are fourfold. First, input prices that are too high can give rise to inefficient market foreclosure and input prices that are too low can create incentives for non-price discrimination. Second, there is a range of non-exclusionary input prices that simultaneously protects against both inefficient foreclosure and sabotage. Third, the safe harbor range of downstream to upstream “price-cost” margin ratios is decreasing in the degree of product homogeneity and approaches a single ratio in the limit as the products become perfectly homogeneous. This single margin ratio preserves equality between the VIP’s wholesale and retail “price-cost” margins. Fourth, a key finding for competition policy is that the bounds of non-exclusionary input prices are markedly wider under Bertrand-Nash competition than they are under Stackelberg competition. Hence, it is critical that the antitrust and regulatory authorities understand the nature of the industry competition so that rules governing permissible conduct are properly calibrated to yield efficient outcomes.

The remainder of the chapter is outlined as follows. The notation and definitions are

introduced in section 3.2. In Section 3.3 we develop the formal model and main findings. Section 3.4 describes the policy applications of our paper in the telecommunications industry. Section 3.5 concludes. The proofs of all formal results are contained in the Appendix B.

3.2 Notation and Definitions

There is a single VIP with a monopoly in the upstream input market and a single independent downstream provider. For the VIP and the independent provider, the downstream inverse demand functions are given by $P^V(Q^V, Q^I)$ and $P^I(Q^I, Q^V)$, where Q^i , $i = V$ and I denote the downstream outputs for the VIP and the independent rival, respectively. The downstream products supplied by the VIP and the independent downstream provider are imperfect substitutes so that $P_{Q^i}^i < P_{Q^j}^i < 0$ for $i, j = V$ and I , $i \neq j$, where the subscripts denote partial derivatives. The price and constant marginal cost of the input supplied by the VIP are denoted by w and c , respectively. The production technology is fixed-coefficient that implies the production of each unit of downstream output requires one unit of the VIP-supplied input and one unit of a complementary input. Each unit of the complementary input costs s^i , $i = V$ and I . Let $d > 0$ denote the increment by which the VIP raises the per-unit cost of its rival through sabotage. Finally, let $C(d)$ denote the VIP's cost of sabotage, with $C(0) = 0$, $C'(0) = 0$, $C''(d) > 0$, and $C'''(d) > 0 \forall d > 0$.

The profit functions for the VIP and the independent rival satisfy standard regularity conditions that ensure a unique optimum and are given, respectively, by

$$\Pi^V = Q^I(w - c) + Q^V(P^V - c - s^V) - C(d) \quad (3.1)$$

and

$$\Pi^I = Q^I(P^I - w - s^I - d). \quad (3.2)$$

Definition 3.1. (Displacement ratio):

The displacement ratio (σ) is the absolute value of the change in the independent rival's output associated with a one-unit increase in the VIP's output ([Armstrong *et al.*, 1996](#)).

3.3 Formal model

The VIP is assumed to be the leader and the rival is the follower in a Stackelberg setting that permits only non-discriminatory, uniform prices. A formal statement of the VIP's problem [V-P] is given by

$$\begin{aligned} \underset{\{Q^V, Q^I, d\}}{\text{Maximum}} \Pi^V &= Q^I(w - c) + Q^V(P^V - c - s^V) - C(d) \\ \text{subject to : } Q^I &\in \underset{Q^I}{\text{argmax}} \Pi^I = Q^I(P^I - w - s^I - d), \end{aligned} \quad (3.3)$$

where $P^V \geq 0$, $P^I \geq 0$, $d \geq 0$.

The incentive compatibility constraint in [V-P] employs the first-order approach to model the rival's profit-maximizing choice of Q^I . Thus,

$$\frac{\partial \Pi^I}{\partial Q^I} = (P^I - w - s^I - d) + Q^I \left(\frac{\partial P^I}{\partial Q^I} - \frac{\partial w}{\partial P^V} \frac{\partial P^V}{\partial Q^I} \right) = 0. \quad (3.4)$$

The Lagrangian for this problem is given by

$$\begin{aligned} L &= Q^I(w - c) + Q^V(P^V - c - s^V) - C(d) \\ &+ \lambda \left[(P^I - w - s^I - d) + Q^I \left(\frac{\partial P^I}{\partial Q^I} - \frac{\partial w}{\partial P^V} \frac{\partial P^V}{\partial Q^I} \right) \right], \end{aligned} \quad (3.5)$$

where λ is the Lagrange multiplier.

An input price that is too low relative to the output price can give rise to market exclusion in the form of non-price discrimination and underscores the need for a price-ceiling (P-C) constraint. This constraint requires that the downstream price for the VIP be no greater than the incremental cost of providing downstream output plus the net contribution foregone

(opportunity cost) in not providing the upstream input.

Definition 3.2. (P-C constraint):

$$P^V \leq c + s^V + \sigma_u(w - c) \Leftrightarrow w \geq c + \sigma_u^{-1}(P^V - c - s^V) \Leftrightarrow r = \frac{P^V - c - s^V}{w - c} \leq \sigma_u. \quad (3.6)$$

This constraint requires that the upper-bound displacement ratio (σ_u) be greater than or equal to the ratio of downstream to upstream “price-cost” margins (r).

The VIP is generally required by the antitrust or regulatory authority to satisfy a price-floor (P-F) constraint that ensures its retail price is no lower than the sum of the direct cost and the opportunity cost of providing the downstream output. The opportunity cost is measured in terms of the net revenue from input sales foregone when the VIP supplies one additional unit of output.

Definition 3.3. (P-F constraint):

$$P^V \geq c + s^V + \sigma_l(w - c) \Leftrightarrow w \leq c + \sigma_l^{-1}(P^V - c - s^V) \Leftrightarrow r = \frac{P^V - c - s^V}{w - c} \geq \sigma_l. \quad (3.7)$$

The lower-bound displacement ratio (σ_l) is derived from the rival’s reaction function because it measures the output of the rival that is displaced when the VIP supplies an additional unit of output. This constraint requires that the lower-bound displacement ratio be less than or equal to the ratio of downstream to upstream “price-cost” margins.

As formalized in the following definition, binding P-C and P-F constraints define the lower bound input price (\underline{w}) and the upper bound input price (\bar{w}), respectively.

Definition 3.4. (Lower/upper-bound input prices and margin ratios):

a) The lower-bound input price and upper-bound margin ratio are given by

$$\underline{w}_{(\sigma_u^{-1})} = c + \sigma_u^{-1}(P^V - c - s^V) \Leftrightarrow \bar{r} = \frac{P^V - c - s^V}{w - c} = \sigma_u. \quad (3.8)$$

b) The upper-bound input price and lower-bound margin ratio are given by

$$\bar{w}_{(\sigma_l^{-1})} = c + \sigma_l^{-1}(P^V - c - s^V) \Leftrightarrow \underline{r} = \frac{P^V - c - s^V}{w - c} = \sigma_l. \quad (3.9)$$

In the analysis that follows, we specify the input price as $w = c + k(P^V - c - s^V)$, where k is the relevant inverse displacement ratio.

Lemma 3.1. The upper-bound displacement ratio is

$$\sigma_u = \left| \frac{\partial Q^I}{\partial Q^V} \right| = \frac{\partial P^V}{\partial Q^V} / \frac{\partial P^V}{\partial Q^I}. \quad (3.10)$$

Lemma 3.2. The lower-bound displacement ratio is

$$\sigma_l = \left| \frac{\partial Q^I}{\partial Q^V} \right| = \frac{2 \left| \frac{\partial P^V}{\partial Q^V} + Q^I \frac{\partial^2 P^V}{\partial Q^I \partial Q^V} \right|}{\left[\frac{\partial P^I}{\partial Q^V} - 2 \frac{\partial P^V}{\partial Q^I} - Q^I \left(\frac{\partial^2 P^V}{(\partial Q^I)^2} - \frac{\partial^2 P^I}{\partial Q^I \partial Q^V} \right) - \right.} \cdot$$

$$\left[8 \frac{\partial P^I}{\partial Q^I} \frac{\partial P^V}{\partial Q^V} - 4 \frac{\partial P^I}{\partial Q^V} \frac{\partial P^V}{\partial Q^I} + \left(\frac{\partial P^I}{\partial Q^V} \right)^2 + 4 \left(\frac{\partial P^V}{\partial Q^I} \right)^2 + \left(Q^I \frac{\partial^2 P^V}{\partial Q^I \partial Q^V} \right)^2 + \right]^{1/2}$$

$$\left[\begin{array}{c} \frac{\partial P^I}{\partial Q^V} \frac{\partial^2 P^I}{\partial Q^I \partial Q^V} - \frac{\partial P^I}{\partial Q^V} \frac{\partial^2 P^V}{(\partial Q^I)^2} \\ + 2 \frac{\partial P^V}{\partial Q^V} \frac{\partial^2 P^I}{(\partial Q^I)^2} - 2 \frac{\partial P^V}{\partial Q^I} \frac{\partial^2 P^I}{\partial Q^I \partial Q^V} \\ + 4 \frac{\partial P^I}{\partial Q^I} \frac{\partial^2 P^V}{\partial Q^I \partial Q^V} + 2 \frac{\partial P^V}{\partial Q^I} \frac{\partial^2 P^V}{(\partial Q^I)^2} - \\ Q^I \left(\frac{\partial^2 P^I}{\partial Q^I \partial Q^V} \frac{\partial^2 P^V}{(\partial Q^I)^2} - 2 \frac{\partial^2 P^I}{(\partial Q^I)^2} \frac{\partial^2 P^V}{\partial Q^I \partial Q^V} \right) \end{array} \right] \right]^{1/2} \quad (3.11)$$

Definition 3.5. (Product homogeneity):

The degree of product homogeneity is given by $\theta = \frac{\sigma_l}{\sigma_u} \in (0, 1)$.

Assumption 3.1. The displacement ratios, σ_i , $i = l$ and u , are constants.

The first proposition establishes that the VIP does not engage in non-price discrimination for any input price that satisfies the P-C constraint. This implies that the VIP does not engage in non-price discrimination for any margin ratio that is no greater than the upper-bound margin ratio.

Proposition 3.1. At the solution to [V-P], $d^* = 0 \forall k \geq \sigma_u^{-1} \Rightarrow \forall w \geq \underline{w} \Rightarrow \forall r \leq \bar{r}$.

The second proposition establishes that the VIP engages in non-price discrimination or sabotage when the input price is strictly less than the lower-bound input price, and consequently when the margin ratio is strictly greater than the upper-bound margin ratio.

Proposition 3.2. At the solution to [V-P], $d^* > 0 \forall k < \sigma_u^{-1} \Rightarrow \forall w < \underline{w} \Rightarrow \forall r > \bar{r}$.

The third proposition establishes that the VIP does not engage in either type of market exclusion for any input price (margin ratio) that satisfies both the P-F and P-C constraints.

Proposition 3.3. The VIP does not engage in market exclusion $\forall k \in [\sigma_u^{-1}, \sigma_l^{-1}] \Rightarrow \forall w \in [\underline{w}, \bar{w}] \Rightarrow \forall r \in [\underline{r}, \bar{r}]$.

Figure 3.1 shows that market exclusion takes the form of inefficient foreclosure and sabotage when $w > \bar{w} \Rightarrow r < \underline{r}$ and when $w < \underline{w} \Rightarrow r > \bar{r}$, respectively. Hence, the admissible range of non-exclusionary input prices and margin ratios is defined by $w \in [\underline{w}, \bar{w}] \Rightarrow r \in [\underline{r}, \bar{r}]$.

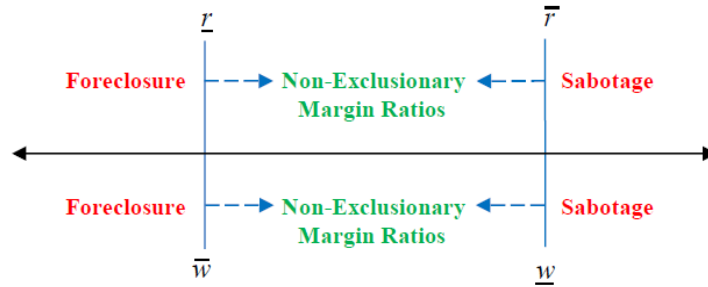


Figure 3.1: Range of non-exclusionary input prices (margin ratios).

The following proposition shows that the safe harbor range of margin ratios reduces to a single ratio in the limit as the degree of product homogeneity approaches unity (i.e., perfectly homogeneous).

Proposition 3.4. In the limit as products become perfectly homogeneous ($\theta \rightarrow 1$), $\underline{r} \rightarrow \bar{r}$.

Corollary 3.1. In the limit as products become perfectly homogeneous ($\theta \rightarrow 1$), the non-exclusionary margin ratio is unique and satisfies the “equal-margin rule.”

The “equal-margin rule” satisfies $P^V - c - s^V = w - c$.

Example 3.1. The inverse demand functions for the VIP and the independent rival are specified as $P^{V/I}(Q^{V/I}, Q^{I/V}) = 20 - 2Q^{V/I} - 2gQ^{I/V}$, where $g \in (0, 1)$. Further, $s^V = s^I = c = 1$. Tables 3.1 and 3.2 presents the numerical simulations.¹

The numerical simulations shown in Tables 3.1 and 3.2 confirm the theoretical findings and provide additional insights. First, consistent with Proposition 3.4, the range of non-exclusionary margin ratios ($\bar{r} - \underline{r}$) approaches zero as θ approaches unity. Second, consistent with Corollary 3.1, in the limit as $\theta \rightarrow 1$, $P^V - c - s^V = w - c$, which is the equal margin rule. Third, as the degree of product homogeneity increases, each additional unit of the rival’s output displaces a larger amount of the VIP output. Hence, \underline{w} increases with the degree of product homogeneity to compensate the VIP for this displaced output.² Fourth, the rival earns positive profits when the services of the VIP and the rival are sufficiently differentiated, but the rival’s profits approach zero as the services become increasingly homogeneous at the upper-bound input price, \bar{w} . The more homogeneous are the product offerings, the greater the ability of the VIP to appropriate the rival’s profits through the input price. Fifth, recognize that the monopoly outcome in this example is $P^V = 11$, $Q^V = 4.5$ and $\Pi^V = 40.5$. The VIP sets the lower-bound input price, \underline{w} , to induce the rival follower to set $P_{(\sigma_u^{-1})}^I = 11$,

¹Appealing to Lemmas 3.1 and 3.2, it is readily shown for this example that $\sigma_u^{-1} = g$ and $\sigma_l^{-1} = \left| \frac{2g - \sqrt{32 + 4g^2}}{-4} \right|$. In the limit, as $\theta \rightarrow 1$, $g \rightarrow 1$, which implies $\sigma_u^{-1} \rightarrow 1$ and $\sigma_l^{-1} \rightarrow 1$.

²Recall that \underline{w} is defined as that input price at which the VIP is indifferent between selling an additional unit of downstream output and selling an additional unit of the input to its rival.

Table 3.1: Numerical simulation results.

g	σ_u^{-1}	σ_l^{-1}	θ	$\underline{w}_{(\sigma_u^{-1})}$	$P_{(\sigma_u^{-1})}^V$	$P_{(\sigma_u^{-1})}^I$	$Q_{(\sigma_u^{-1})}^V$	$Q_{(\sigma_u^{-1})}^I$	$\Pi_{(\sigma_u^{-1})}^V$	$\Pi_{(\sigma_u^{-1})}^I$	\bar{r}
0.25	0.25	1.29	0.19	3.25	11.00	11.00	3.60	3.60	40.50	24.30	4.00
0.50	0.50	1.19	0.42	5.50	11.00	11.00	3.00	3.00	40.50	13.50	2.00
0.60	0.60	1.15	0.52	6.40	11.00	11.00	2.81	2.81	40.50	10.13	1.67
0.70	0.70	1.11	0.63	7.30	11.00	11.00	2.65	2.65	40.50	7.15	1.43
0.80	0.80	1.07	0.75	8.20	11.00	11.00	2.50	2.50	40.50	4.50	1.25
0.90	0.90	1.03	0.87	9.10	11.00	11.00	2.37	2.37	40.50	2.13	1.11
0.95	0.95	1.02	0.93	9.55	11.00	11.00	2.31	2.31	40.50	1.04	1.05
0.97	0.97	1.01	0.96	9.73	11.00	11.00	2.28	2.28	40.50	0.62	1.03

Table 3.2: Numerical simulation results.

g	σ_u^{-1}	σ_l^{-1}	θ	$\bar{w}_{(\sigma_l^{-1})}$	$P_{(\sigma_l^{-1})}^V$	$P_{(\sigma_l^{-1})}^I$	$Q_{(\sigma_l^{-1})}^V$	$Q_{(\sigma_l^{-1})}^I$	$\Pi_{(\sigma_l^{-1})}^V$	$\Pi_{(\sigma_l^{-1})}^I$	\underline{r}
0.25	0.25	1.29	0.19	11.33	9.98	14.49	4.61	1.60	53.32	3.47	0.77
0.50	0.50	1.19	0.42	10.84	10.29	13.00	4.14	1.43	48.39	1.66	0.84
0.60	0.60	1.15	0.52	10.66	10.43	12.51	3.97	1.37	46.62	1.17	0.87
0.70	0.70	1.11	0.63	10.48	10.57	12.07	3.81	1.30	44.94	0.76	0.90
0.80	0.80	1.07	0.75	10.31	10.71	11.67	3.65	1.24	43.37	0.44	0.93
0.90	0.90	1.03	0.87	10.15	10.85	11.32	3.51	1.18	41.89	0.19	0.97
0.95	0.95	1.02	0.93	10.08	10.93	11.15	3.44	1.15	41.19	0.09	0.98
0.97	0.97	1.01	0.96	10.05	10.96	11.09	3.41	1.14	40.91	0.05	0.99

which, in turn, sustains monopoly profits, the sum of upstream and downstream profits, for the VIP for all value of θ . Sixth, as $\theta \rightarrow 1$, $\Pi_{(\sigma_l^{-1})}^V \rightarrow 40.5$, the monopoly level of profits. To understand this result, recognize that for low values of θ , which corresponds to high levels of product differentiation, the VIP earns in excess of monopoly profits by using the input price to extract a share of the rival's profits from a segment of the market that only marginally cannibalizes the VIP's profits. Conversely, as θ grows large, the degree of cannibalization increases, limiting the VIP's profits and retail price to monopoly levels.

Figure 3.2 illustrates the bounds of the non-exclusionary input prices for Bertrand-Nash competition and Stackelberg competition. A key observation is that the bounds of the non-exclusionary input prices are markedly greater under Bertrand-Nash competition. This observation underscores the importance of identifying the correct market structure. Specifically, if the regulator believes that competition is Bertrand-Nash when it is actually Stackelberg

there is a greater likelihood of unwittingly permitting exclusionary behavior. In practice, the regulator may not have perfect information regarding the nature of competition in the industry. Hence, in the presence of imperfect information, the regulator may be presented with the prospect of making a Type I error³ or Type II error⁴ with regard to calibrating the permissible bounds on input prices. The objective for the regulator would then be one of calibrating the allowable range of input prices so as to minimize the social cost of “being wrong.” This is a question that lies beyond the extant analysis, but is an interesting topic for future research.

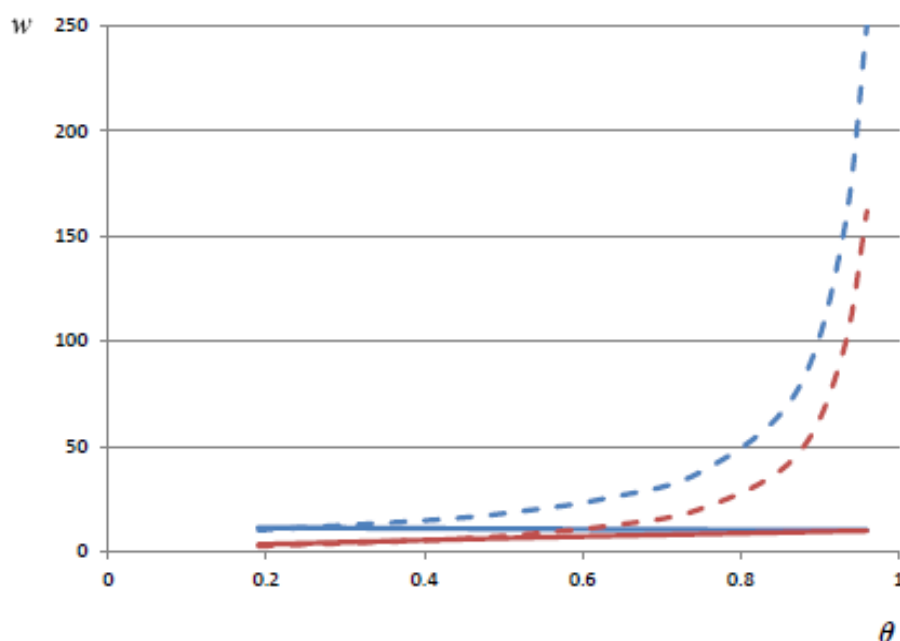


Figure 3.2: Upper-bound (blue) and lower-bound (red) input prices under Bertrand- Nash (dashed lines) competition and Stackelberg (solid lines) competition.

3.4 Application: Telecommunications

Following the passage of the 1996 Telecommunications Act, allegations of both forms of exclusionary behavior (foreclosure and non-price discrimination) were made against incumbent

³Mistakenly prohibiting procompetitive activities.

⁴Mistakenly permitting anticompetitive activities.

telecommunications providers. As an example for foreclosure, in the Linkline case, the incumbent VIP, AT&T, was accused of unlawfully squeezing its rivals' profit margins through a combination of high input (wholesale) prices and low retail prices in the market for digital subscriber-line (DSL). This squeezed the profits of the incumbent's rivals and purportedly caused some of them to exit the market ([Squeo, 2004](#)). As the Supreme Court observed:

“The plaintiffs in this case, respondents here, allege that a competitor subjected them to a “price squeeze” in violation of § 2 of the Sherman Act. They assert that such a claim can arise when a vertically integrated firm sells inputs at wholesale and also sells finished goods or services at retail. If that firm has power in the wholesale market, it can simultaneously raise the wholesale price of inputs and cut the retail price of the finished good. This will have the effect of “squeezing” the profit margins of any competitors in the retail market.”⁵

In the Trinko case, Verizon, the incumbent VIP, was accused of raising the cost of its rival by degrading the quality of access provided to that rival, and therefore making it more difficult to compete against Verizon.⁶ This, of course, is a form of non-price discrimination or sabotage that can arise when wholesale prices are “too low” relative to retail prices.

“The complaint . . . alleged that Verizon had filled rivals' orders on a discriminatory basis as part of an anticompetitive scheme to discourage customers from becoming or remaining customers of competitive LECs, thus impeding the competitive LECs' ability to enter and compete in the market for local telephone service.”⁷

Regulators in the telecommunications industry have recognized the importance of being proactive in preventing a price squeeze through price-floor constraints. However, to date they

⁵[Pacific Bell Telephone Co. v. Linkline Communications \(2009\)](#).

⁶In a similar context, the antitrust division of the U.S. Department of Justice announced that it was investigating whether the data caps instituted by AT&T and Comcast are intended to discourage their customers from switching to online video providers such as Netflix and Hulu. See [Catan & Schatz, 2012](#).

⁷[Verizon Communications Inc. v. Law Offices of Curtis V. Trinko \(2004\)](#).

have not taken into consideration the need to prevent non-price discrimination through price-ceiling constraints. While the preponderance of competitor complaints about the behavior of the incumbent providers concern allegations of non-price discrimination, the competitors' regulatory advocacy focuses almost exclusively on securing the lowest possible prices for essential inputs.⁸ And yet, lower input prices can lead to higher costs for competitors by encouraging the VIPs to engage in non-price discrimination. Therefore, the results of our study can be applied to assist regulatory authorities to recognize different possible forms of market exclusion in a particular industry and seek for proper solutions to prevent inefficient completion.

3.5 Conclusions

This paper explores the relationship between upstream input prices and downstream market exclusion under a Stackelberg quantity competition framework. Inefficient foreclosure can arise when input prices are “too high” and sabotage can arise when input prices are “too low.” Upper and lower-bound displacement ratios are used to generate a range of non-exclusionary input prices. The admissible range of the ratio of downstream to upstream margins is decreasing in the degree of product homogeneity and reduces to a single ratio in the limit as the products become perfectly homogeneous. This single margin ratio preserves equality between the VIP's wholesale and retail “price-cost” margins. A key finding for competition policy is that the bounds of non-exclusionary input prices are markedly wider under Bertrand-Nash competition than they are under Stackelberg competition. Hence, it is critical that the antitrust and regulatory authorities understand the nature of the industry competition so that rules governing permissible conduct are properly calibrated to yield efficient outcomes. Regulators may choose to complement price-floor constraint with a price-ceiling constraint in order to prevent both forms of market exclusion. However, whether they should take

⁸See, for example, [Kahn *et al.*, 1999](#), [Verizon Communications Inc. v. Federal Communications Commission \(2002\)](#), and [Tardiff, 2002](#).

proactive measures to protect against these types of market exclusion, while important and interesting, is beyond the scope of this chapter.

It would be interesting for future research to further explore the range of non-exclusionary input prices under more general technologies, non-constant displacement ratios, and different game structures. Changing the structure of the analysis is expected to have a quantitative effect on the range of non-exclusionary input prices, but would not alter the key qualitative implications for competition policy.

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Appendix A

Chapter 2 Proofs

Proof of Lemma 2.1

$$dQ^V(P^V, P^I) = \frac{\partial Q^V}{\partial P^V} dP^V + \frac{\partial Q^V}{\partial P^I} dP^I. \quad (\text{A.1})$$

Set $dQ^V = 1$ and $dP^V = 0$ yields

$$dP^I = \left(\frac{\partial Q^V}{\partial P^I} \right)^{-1}. \quad (\text{A.2})$$

Also,

$$dQ^I(P^I, P^V) = \frac{\partial Q^I}{\partial P^V} dP^V + \frac{\partial Q^I}{\partial P^I} dP^I. \quad (\text{A.3})$$

Set $dP^V = 0$, and substituting for dP^I yields

$$|dQ^I| = \left| \frac{\partial Q^I}{\partial P^I} dP^I \right| = \left| \frac{\partial Q^I}{\partial P^I} / \frac{\partial Q^V}{\partial P^I} \right|. \quad (\text{A.4})$$

Proof of Lemma 2.2

$$dQ^V(P^V, P^I) = \frac{\partial Q^V}{\partial P^V} dP^V + \frac{\partial Q^V}{\partial P^I} dP^I. \quad (\text{A.5})$$

Set $dQ^V = 1$ and $dP^I = 0$ yields

$$dP^V = \left(\frac{\partial Q^V}{\partial P^V} \right)^{-1}. \quad (\text{A.6})$$

Also,

$$dQ^I(P^I, P^V) = \frac{\partial Q^I}{\partial P^V} dP^V + \frac{\partial Q^I}{\partial P^I} dP^I. \quad (\text{A.7})$$

Recognizing that $dP^I = \frac{\partial P^I}{\partial P^V} dP^V$ and substituting for dP^V yields

$$|dQ^I| = \left| \frac{\partial Q^I}{\partial P^V} dP^V + \frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial P^V} dP^V \right| = \left| \left(\frac{\partial Q^I}{\partial P^V} + \frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial P^V} \right) \frac{\partial Q^V}{\partial P^V} \right|. \quad (\text{A.8})$$

Proof of Lemma 2.3

Totally differentiating (2.9) and (2.10) with respect to d yields the linear system $A.X = B$:

$$\overbrace{\begin{bmatrix} \underbrace{\frac{\partial^2 \Pi^V}{(\partial P^V)^2}}_{-} & \underbrace{\frac{\partial^2 \Pi^V}{\partial P^V \partial P^I}}_{+} \\ \underbrace{\frac{\partial^2 \Pi^I}{\partial P^I \partial P^V}}_{+} & \underbrace{\frac{\partial^2 \Pi^I}{(\partial P^I)^2}}_{-} \end{bmatrix}}^A \overbrace{\begin{bmatrix} \frac{\partial P^V}{\partial d} \\ \frac{\partial P^I}{\partial d} \end{bmatrix}}^X = \overbrace{\begin{bmatrix} 0 \\ \frac{\partial Q^I}{\partial P^I} \end{bmatrix}}^B. \quad (\text{A.9})$$

Sufficient second-order conditions for a maximum require that $\frac{\partial^2 \Pi^V}{(\partial P^V)^2} < 0$, $\frac{\partial^2 \Pi^I}{(\partial P^I)^2} < 0$

and $|A| = \frac{\partial^2 \Pi^V}{(\partial P^V)^2} \frac{\partial^2 \Pi^I}{(\partial P^I)^2} - \frac{\partial^2 \Pi^V}{\partial P^V \partial P^I} \frac{\partial^2 \Pi^I}{\partial P^I \partial P^V} > 0$.

Appealing to Cramer's rule yields

$$\frac{\partial P^I}{\partial d} = \frac{1}{|A|} \begin{vmatrix} \frac{\partial^2 \Pi^V}{(\partial P^V)^2} & 0 \\ \frac{\partial^2 \Pi^I}{\partial P^I \partial P^V} & \frac{\partial Q^I}{\partial P^I} \end{vmatrix} = \frac{\frac{\partial^2 \Pi^V}{(\partial P^V)^2} \frac{\partial Q^I}{\partial P^I}}{|A|} > 0 \quad (\text{A.10})$$

and

$$\frac{\partial P^V}{\partial d} = \frac{1}{|A|} \begin{vmatrix} 0 & \frac{\partial^2 \Pi^V}{\partial P^V \partial P^I} \\ \frac{\partial Q^I}{\partial P^I} & \frac{\partial^2 \Pi^I}{(\partial P^I)^2} \end{vmatrix} = \frac{-\frac{\partial^2 \Pi^V}{\partial P^V \partial P^I} \frac{\partial Q^I}{\partial P^I}}{|A|} > 0. \quad (\text{A.11})$$

since $\frac{\partial^2 \Pi^V}{\partial P^V \partial P^I} > 0$ when prices are strategic complements.

Proof of Proposition 2.1

From (2.1), the necessary first-order condition for d is given by

$$\begin{aligned} \Pi_d^V : & \left[\frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial d} + \frac{\partial Q^I}{\partial P^V} \frac{\partial P^V}{\partial d} \right] (w - c) + Q^I \frac{\partial w}{\partial P^V} \frac{\partial P^V}{\partial d} + Q^V \frac{\partial P^V}{\partial d} \\ & + \left[\frac{\partial Q^V}{\partial P^V} \frac{\partial P^V}{\partial d} + \frac{\partial Q^V}{\partial P^I} \frac{\partial P^I}{\partial d} \right] (P^V - c - s^V) - C'(d) \leq 0 \end{aligned} \quad (\text{A.12})$$

and

$$d(\Pi_d^V) = 0. \quad (\text{A.13})$$

Let $k = \Delta + \sigma_u^{-1}$, where $\Delta \geq 0$. Substituting for $\sigma_u^{-1} = \left| \frac{\partial Q^V}{\partial P^I} / \frac{\partial Q^I}{\partial P^I} \right| = - \left(\frac{\partial Q^V}{\partial P^I} / \frac{\partial Q^I}{\partial P^I} \right)$ along with $\underline{w} = c + (\Delta + \sigma_u^{-1})(P^V - c - s^V)$ and (A.12) yields

$$\begin{aligned} & \left(\frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial d} + \frac{\partial Q^I}{\partial P^V} \frac{\partial P^V}{\partial d} \right) (\Delta + \sigma_u^{-1})(P^V - c - s^V) + Q^I (\Delta + \sigma_u^{-1}) \frac{\partial P^V}{\partial d} \\ & + \left(\frac{\partial Q^V}{\partial P^V} \frac{\partial P^V}{\partial d} + \frac{\partial Q^V}{\partial P^I} \frac{\partial P^I}{\partial d} \right) (P^V - c - s^V) + Q^V \frac{\partial P^V}{\partial d} - C'(d) \leq 0. \end{aligned} \quad (\text{A.14})$$

Rewrite (A.14) in the following form:

$$\begin{aligned} & \left[\Delta \left(\frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial d} \right) + \left(\frac{\partial Q^I}{\partial P^V} \frac{\partial P^V}{\partial d} \right) \left(\Delta - \left(\frac{\partial Q^V}{\partial P^I} / \frac{\partial Q^I}{\partial P^I} \right) \right) \right] (P^V - c - s^V) \\ & + Q^I \left(\Delta - \left(\frac{\partial Q^V}{\partial P^I} / \frac{\partial Q^I}{\partial P^I} \right) \right) \frac{\partial P^V}{\partial d} + \frac{\partial Q^V}{\partial P^V} \frac{\partial P^V}{\partial d} (P^V - c - s^V) + Q^V \frac{\partial P^V}{\partial d} - C'(d) \leq 0. \end{aligned} \quad (\text{A.15})$$

At an optimum for (2.1), $\Pi_{P^V}^V = 0$ or

$$\begin{aligned} & \frac{\partial Q^I}{\partial P^V} \left(\Delta - \left(\frac{\partial Q^V}{\partial P^I} / \frac{\partial Q^I}{\partial P^I} \right) \right) (P^V - c - s^V) + Q^I \left(\Delta - \left(\frac{\partial Q^V}{\partial P^I} / \frac{\partial Q^I}{\partial P^I} \right) \right) \\ & + \frac{\partial Q^V}{\partial P^V} (P^V - c - s^V) + Q^V = 0. \end{aligned} \quad (\text{A.16})$$

Substituting (A.16) into (A.15) and appealing to Lemma 2.3 yields

$$\Delta \left(\underbrace{\frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial d}}_{-} \right) \underbrace{(P^V - c - s^V)}_{+} - C'(d) \leq 0. \quad (\text{A.17})$$

Since $\Delta \geq 0$, (A.17) implies that $d^* = 0$ by complementary slackness from (A.12)-(A.13).

Proof of Proposition 2.2

From (A.17), $\Delta < 0$ implies that $-C'(d) < 0 \Rightarrow d^* > 0$.

Proof of Proposition 2.3

By Proposition 2.1, $d^* = 0 \ \forall k \geq \sigma_u^{-1} \Rightarrow w \geq \underline{w}$.

By Proposition 2.2, $d^* > 0 \ \forall k < \sigma_u^{-1} \Rightarrow w < \underline{w}$. If $k > \sigma_l^{-1}$ then $w > \bar{w}$ and the P-F constraint is violated. The result follows.

Proof of Proposition 2.4

Perfect product homogeneity implies

$$|Q_{PI}^I| = |Q_{PI}^V| \Rightarrow \left| \frac{\partial Q^I}{\partial P^I} \right| = \left| \frac{\partial Q^V}{\partial P^I} \right| \quad (\text{A.18})$$

and

$$|Q_{PV}^I| = |Q_{PV}^V| \Rightarrow \left| \frac{\partial Q^I}{\partial P^V} + \frac{\partial Q^I}{\partial P^I} \frac{\partial P^I}{\partial P^V} \right| = \left| \frac{\partial Q^V}{\partial P^V} \right|. \quad (\text{A.19})$$

where the subscripts denote partial derivatives.

(A.18) and (A.19) imply that as $\theta \rightarrow 1 \Rightarrow \sigma_l \rightarrow \sigma_u \approx 1$ and $\underline{r} \rightarrow \bar{r}$.

Proof of Corollary 2.1

$\theta \rightarrow 1 \Rightarrow \sigma_l \rightarrow \sigma_u \approx 1$. Satisfaction of the P-C and P-F constraints requires that $\frac{P^V - c - s^V}{w - c} \leq 1$ and $\frac{P^V - c - s^V}{w - c} \geq 1 \Rightarrow \frac{P^V - c - s^V}{w - c} = 1 \Rightarrow P^V - c - s^V = w - c$, the “equal-margin rule.”

Proof of Proposition 2.5

Substituting for w in (2.10), appealing to (2.12) and solving for P^I yields

$$P^I = \frac{a^I + b^I (c + d + s^I + k(P^V - c - s^V)) + P^V g^I}{2b^I}. \quad (\text{A.20})$$

Substituting for w and P^I along with solving (2.9) and (2.10) simultaneously for the linear system in (2.11) and (2.12) yields the Nash-equilibrium prices in (2.13) and (2.14).

Proof of Proposition 2.6

Substituting (2.11)-(2.14) into (2.1) yields the VIP’s reduced-form profit function in (2.15).

Appendix B

Chapter 3 Proofs

Proof of Lemma 3.1

Recognizing that $w = c + k(P^V - c - s^V)$ and $\frac{\partial w}{\partial P^V} = k$, from (3.1), the first-order condition for Q^V is given by

$$\frac{\partial \Pi^V}{\partial Q^V} = P^V - c - s^V + \frac{\partial P^V}{\partial Q^V}(kQ^I + Q^V) = 0. \quad (\text{B.1})$$

Imposing the condition that the VIP's profits are non-decreasing in the output of the rival implies

$$\frac{\partial \Pi^V}{\partial Q^I} = kQ^I \frac{\partial P^V}{\partial Q^I} + k(P^V - c - s^V) + Q^V \frac{\partial P^V}{\partial Q^I} \geq 0. \quad (\text{B.2})$$

Rewrite (B.1) in the following form:

$$kQ^I + Q^V = -\frac{(P^V - c - s^V)}{\frac{\partial P^V}{\partial Q^V}}. \quad (\text{B.3})$$

Substituting (B.3) into (B.2) yields

$$(P^V - c - s^V) \left(k - \frac{\partial P^V}{\partial Q^I} / \frac{\partial P^V}{\partial Q^V} \right) \geq 0. \quad (\text{B.4})$$

(B.4) implies that

$$k = \sigma^{-1} \geq \frac{\partial P^V}{\partial Q^I} / \frac{\partial P^V}{\partial Q^V} \Rightarrow \sigma_u = \frac{\partial P^V}{\partial Q^V} / \frac{\partial P^V}{\partial Q^I}. \quad (\text{B.5})$$

Proof of Lemma 3.2

Recognizing that $w = c + k(P^V - c - s^V)$, $\frac{\partial w}{\partial P^V} = k$, and $d = 0$, (3.4) yields

$$\frac{\partial \Pi^V}{\partial Q^I} = (P^I - w - s^I) + Q^I \left(\frac{\partial P^I}{\partial Q^I} - k \frac{\partial P^V}{\partial Q^I} \right) = 0. \quad (\text{B.6})$$

From (B.6), the rival's reaction function can be written as

$$Q^I = - \frac{(P^I - w - s^I)}{\frac{\partial P^I}{\partial Q^I} - k \frac{\partial P^V}{\partial Q^I}}. \quad (\text{B.7})$$

Differentiating (B.7) with respect to Q^V yields

$$\begin{aligned} \frac{\partial Q^I}{\partial Q^V} = & - \frac{\left(\frac{\partial P^I}{\partial Q^V} + \frac{\partial P^I}{\partial Q^I} \frac{\partial Q^I}{\partial Q^V} - k \left(\frac{\partial P^V}{\partial Q^V} + \frac{\partial P^V}{\partial Q^I} \frac{\partial Q^I}{\partial Q^V} \right) \right)}{\frac{\partial P^I}{\partial Q^I} - k \frac{\partial P^V}{\partial Q^I}} \\ & + \frac{(P^I - w - s^I) \left(\frac{\partial^2 P^I}{\partial Q^I \partial Q^V} + \frac{\partial^2 P^I}{(\partial Q^I)^2} \frac{\partial Q^I}{\partial Q^V} - k \left(\frac{\partial^2 P^V}{\partial Q^I \partial Q^V} + \frac{\partial^2 P^V}{(\partial Q^I)^2} \frac{\partial Q^I}{\partial Q^V} \right) \right)}{\left(\frac{\partial P^I}{\partial Q^I} - k \frac{\partial P^V}{\partial Q^I} \right)^2}. \end{aligned} \quad (\text{B.8})$$

Substituting $(P^I - w - s^I) = -Q^I \left(\frac{\partial P^I}{\partial Q^I} - k \frac{\partial P^V}{\partial Q^I} \right)$ from (B.7), rewrite (B.8) in the following form

$$\left| \frac{\partial Q^I}{\partial Q^V} \right| = \left| \frac{k \frac{\partial P^V}{\partial Q^V} - \frac{\partial P^I}{\partial Q^V} - Q^I \left(\frac{\partial^2 P^I}{\partial Q^I \partial Q^V} - k \frac{\partial^2 P^V}{\partial Q^I \partial Q^V} \right)}{2 \left(\frac{\partial P^I}{\partial Q^I} - k \frac{\partial P^V}{\partial Q^I} \right) + Q^I \left(\frac{\partial^2 P^I}{(\partial Q^I)^2} - k \frac{\partial^2 P^V}{(\partial Q^I)^2} \right)} \right|. \quad (\text{B.9})$$

Recognizing that $k^{-1} = \sigma_l = \left| \frac{\partial Q^I}{\partial Q^V} \right|$ and solving for σ_l yields (3.11).

Proof of Proposition 3.1

From (3.5), the necessary first-order condition for d is given by

$$L_d : -C'(d) + \lambda \left[\frac{\partial^2 \Pi^I}{\partial Q^I \partial d} \right] \leq 0 \quad (\text{B.10})$$

and

$$d(L_d) = 0. \quad (\text{B.11})$$

Recognizing that the term in the brackets is an optimized value function and applying the envelope theorem yields $\frac{\partial^2 \Pi^I}{\partial Q^I \partial d} = -1$. Thus,

$$L_d : -C'(d) - \lambda \leq 0. \quad (\text{B.12})$$

Hence,

$$d^* = \begin{cases} 0 & \text{if } \lambda \geq 0 \\ > 0 & \text{if } \lambda < 0 \end{cases}. \quad (\text{B.13})$$

Recognizing that $w = c + k(P^V - c - s^V)$ and $\frac{\partial w}{\partial P^V} = k$, at an optimum for (3.5), $L_{Q^I} = 0$ or

$$k(P^V - c - s^V) + kQ^I \frac{\partial P^V}{\partial Q^I} + Q^V \frac{\partial P^V}{\partial Q^I} + \lambda \frac{\partial^2 \Pi^I}{(\partial Q^I)^2} = 0, \quad (\text{B.14})$$

where $\frac{\partial^2 \Pi^I}{(\partial Q^I)^2} < 0$.

Substituting (B.3) into (B.14) yields

$$(P^V - c - s^V) \left(k - \frac{\partial P^V}{\partial Q^I} / \frac{\partial P^V}{\partial Q^V} \right) + \lambda \frac{\partial^2 \Pi^I}{(\partial Q^I)^2} = 0. \quad (\text{B.15})$$

Recognizing that $\frac{\partial^2 \Pi^I}{(\partial Q^I)^2} < 0$, if $k \geq \underbrace{\frac{\partial P^V}{\partial Q^I} / \frac{\partial P^V}{\partial Q^V}}_{\sigma_u^{-1}}$, $\lambda \geq 0$ that implies $d^* = 0$ upon appeal to (B.13).

Proof of Proposition 3.2

If $k < \frac{\partial P^V}{\partial Q^I} / \frac{\partial P^V}{\partial Q^V}$, (B.15) implies that $\lambda < 0$ that implies $d^* > 0$ upon appeal to (B.13).

Proof of Proposition 3.3

By Proposition 3.1, $d^* = 0 \ \forall k \geq \sigma_u^{-1} \Rightarrow \forall w \geq \underline{w} \Rightarrow \forall r \leq \bar{r}$. By Proposition 3.2, $d^* > 0 \ \forall k < \sigma_u^{-1} \Rightarrow \forall w < \underline{w} \Rightarrow \forall r > \bar{r}$. If $k > \sigma_l^{-1}$ then $w > \bar{w}$ and the P-F constraint is violated. The result follows.

Proof of Proposition 3.4

Perfect product homogeneity implies

$$\frac{\partial P^I}{\partial Q^I} = \frac{\partial P^V}{\partial Q^I} = \frac{\partial P^I}{\partial Q^V} \quad (\text{B.16})$$

and

$$\frac{\partial P^V}{\partial Q^V} = \frac{\partial P^I}{\partial Q^V} = \frac{\partial P^V}{\partial Q^I}. \quad (\text{B.17})$$

(B.16) and (B.17) imply

$$\frac{\partial P^I}{\partial Q^I} = \frac{\partial P^V}{\partial Q^I} = \frac{\partial P^I}{\partial Q^V} = \frac{\partial P^V}{\partial Q^V}. \quad (\text{B.18})$$

Recognizing that the second-partial derivatives of retail prices are continuous, appealing to Young's Theorem, differentiating (B.18) with respect to Q^I yields

$$\frac{\partial^2 P^I}{(\partial Q^I)^2} = \frac{\partial^2 P^V}{(\partial Q^I)^2} = \frac{\partial^2 P^I}{\partial Q^I \partial Q^V} = \frac{\partial^2 P^V}{\partial Q^I \partial Q^V}. \quad (\text{B.19})$$

Appealing to (B.18), (B.5) yields

$$\sigma_u = \frac{\partial P^V}{\partial Q^V} / \frac{\partial P^V}{\partial Q^I} = 1. \quad (\text{B.20})$$

Appealing to (B.18)-(B.19), (3.11) yields

$$\sigma_l = \left| \frac{\partial Q^I}{\partial Q^V} \right| = \frac{2 \left| \frac{\partial P^I}{\partial Q^I} + Q^I \frac{\partial^2 P^I}{(\partial Q^I)^2} \right|}{\left| -\frac{\partial P^I}{\partial Q^I} - \left[9 \left(\frac{\partial P^I}{\partial Q^I} \right)^2 + 4 \left(Q^I \frac{\partial^2 P^I}{(\partial Q^I)^2} \right)^2 + 12 Q^I \frac{\partial P^I}{\partial Q^I} \frac{\partial^2 P^I}{(\partial Q^I)^2} \right]^{1/2} \right|}. \quad (\text{B.21})$$

Recognizing that $\frac{\partial P^I}{\partial Q^I} = -\alpha$ and $\frac{\partial^2 P^I}{(\partial Q^I)^2} = \beta$, where α and β are non-negative, rewrite (B.21) in the following form yields

$$\sigma_l = \left| \frac{\partial Q^I}{\partial Q^V} \right| = \frac{2 \left| -\alpha + Q^I \beta \right|}{\left| \alpha - [9(-\alpha)^2 + 4(Q^I \beta)^2 - 12 Q^I \alpha \beta]^{1/2} \right|} = \frac{2 \left| -\alpha + Q^I \beta \right|}{\left| \alpha - [(3\alpha - 2 Q^I \beta)^2]^{1/2} \right|} = 1. \quad (\text{B.22})$$

(B.20) and (B.22) imply that as $\theta \rightarrow 1$, $\sigma_l \rightarrow \sigma_u \approx 1$ and $\underline{r} \rightarrow \bar{r}$.

Proof of Corollary 3.1

$\theta \rightarrow 1 \Rightarrow \sigma_l \rightarrow \sigma_u \approx 1$. Satisfaction of the P-C and P-F constraints requires that $\frac{P^V - c - s^V}{w - c} \leq 1$ and $\frac{P^V - c - s^V}{w - c} \geq 1 \Rightarrow \frac{P^V - c - s^V}{w - c} = 1 \Rightarrow P^V - c - s^V = w - c$, the “equal-margin rule.”